## Motion in a Plane

## Practice Problems Solutions

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## 1. Frame the Problem

- Sketch and label a diagram of the motion.

- The equations of motion apply to the problem, since the car is assumed to be moving with constant acceleration.


## Identify the Goal

The car's average acceleration after 4.0 s

## Variables and Constants

Known
$v_{\mathrm{i}}=+6.0 \frac{\mathrm{~m}}{\mathrm{~s}}$
$v_{\mathrm{i}}=+38 \frac{\mathrm{~m}}{\mathrm{~s}}$
$\Delta t=4.0 \mathrm{~s}$

## Strategy

Use the equation for acceleration.

## Calculations

$$
\begin{aligned}
a & =\frac{v_{\mathrm{f}}-v_{\mathrm{i}}}{\Delta t} \\
& =\frac{+38 \frac{\mathrm{~m}}{\mathrm{~s}}-\left(+6.0 \frac{\mathrm{~m}}{\mathrm{~s}}\right)}{4.0 \mathrm{~s}} \\
& =+8.0 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}
\end{aligned}
$$

The Indy race car's acceleration is $+8.0 \mathrm{~m} / \mathrm{s}^{2}$.
Validate
The units in the answer were $\mathrm{m} / \mathrm{s}^{2}$, which is correct for acceleration. The average acceleration is positive, since the velocity of the Indy car was increasing while travelling in the positive direction.

## 2. Frame the Problem

- Sketch and label a diagram of the motion.


|  | $y$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 5 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |
| 3 |  |  |  | , |  |  |
| 2 |  |  | ! |  |  |  |
| - 1 |  |  | ! |  |  |  |
| - 1 |  |  |  |  |  | $t(\mathrm{~s})$ |
| ${ }^{\text {E }} 0$ |  | 1,' | 23 | 3 | 4 |  |
| 1>-1 |  | ! |  |  |  |  |
| - -2 | i |  |  |  |  |  |
| -3 | ! |  |  |  |  |  |
| -4 | , |  |  |  |  |  |
| 5 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |

- The equations of motion apply to the problem, since the acceleration was constant.


## Identify the Goal

The car's acceleration once the driver got it started

## Variables and Constants

Known
$v_{1}=-4.0 \frac{\mathrm{~m}}{\mathrm{~s}}$
$v_{\mathrm{f}}=+3.5 \frac{\mathrm{~m}}{\mathrm{~s}}$
$\Delta t=3.0 \mathrm{~s}$

## Unknown

a

Strategy

- Use the equation for acceleration.
- Let uphill be the positive direction.


## Calculations

$a=\frac{v_{\mathrm{f}}-v_{\mathrm{i}}}{\Delta t}$

$$
\begin{aligned}
& =\frac{+3.5 \frac{\mathrm{~m}}{\mathrm{~s}}-\left(-4.0 \frac{\mathrm{~m}}{\mathrm{~s}}\right)}{3.0 \mathrm{~s}} \\
& =\frac{7.5 \frac{\mathrm{~m}}{\mathrm{~s}}}{3.0 \mathrm{~s}} \\
& =+2.5 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}
\end{aligned}
$$

The car's acceleration once the driver got it started was $2.5 \mathrm{~m} / \mathrm{s}^{2}$ [uphill].
Validate
The units in the answer were $\mathrm{m} / \mathrm{s}^{2}$, which is correct for acceleration. The acceleration is positive, since the velocity of the Indy car was increasing uphill.

## 3. Frame the Problem

- Sketch and label a diagram of the motion.

- The equations of motion apply to the problem, since the acceleration was constant. - The final velocity of the bus will be zero.


## Identify the Goal

The velocity the bus was travelling when the brakes were applied

## Variables and Constants

Known

## Unknown

$a=-8.0 \frac{\mathrm{~m}}{\mathrm{~s}} \quad v_{\mathrm{f}}=0.0 \frac{\mathrm{~m}}{\mathrm{~s}}$ $v_{i}$
$\Delta t=3.0 \mathrm{~s}$

Strategy
Select the equation that relates the initial velocity to the final velocity, acceleration, and time interval.
All of the needed quantities are known, so substitute them into the equation.
Simplify.

## Calculations

$v_{\mathrm{i}}=v_{\mathrm{f}}-a \Delta t$

The bus was travelling at $+24 \mathrm{~m} / \mathrm{s}^{2}$ when the brakes were applied.
Validate
The units in the answer were $\mathrm{m} / \mathrm{s}^{2}$, which is correct for velocity. The initial velocity was positive, which is correct.

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## 4. Frame the Problem

- Make a diagram of the motion of the field hockey player that includes the known variables.

- The equations of motion apply to the problem, since the acceleration was constant.


## Identify the Goal

(a) The distance she travelled
(b) Her acceleration

## Variables and Constants

Known
$v_{\mathrm{i}}=0.0 \frac{\mathrm{~m}}{\mathrm{~s}}$
$v_{\mathrm{f}}=4.0 \frac{\mathrm{~m}}{\mathrm{~s}}$
$\Delta t=2.5 \mathrm{~s}$

## Strategy

Use the equation of motion that relates time, initial velocity, and final velocity to displacement.

All of the needed quantities are known, so substitute them into the equation. Simplify.

## Calculations

$\Delta d=\left(\frac{v_{\mathrm{i}}+v_{f}}{2}\right) \Delta t$
$\Delta d=\left(\frac{0.0 \frac{\mathrm{~m}}{\mathrm{~s}}+4.0 \frac{\mathrm{~m}}{\mathrm{~s}}}{2.0}\right)(2.5 \mathrm{~s})$
$\Delta d=2.0 \frac{\mathrm{~m}}{\mathrm{~s}}(2.5 \mathrm{~s})$
$\Delta d=5.0 \mathrm{~m}$
(a) The distance was 5.0 m . Use the information to find the acceleration.

$$
\begin{aligned}
& a=\frac{\Delta v}{\Delta t}=\frac{v_{2}-v_{1}}{\Delta t} \\
& a=\frac{4.0 \frac{\mathrm{~m}}{\mathrm{~s}}-0.0 \frac{\mathrm{~m}}{\mathrm{~s}}}{2.5 \mathrm{~s}} \\
& a=\frac{4.0 \frac{\mathrm{~m}}{\mathrm{~s}}}{2.5 \mathrm{~s}} \\
& a=1.6 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}
\end{aligned}
$$

(b) Her acceleration is $1.6 \mathrm{~m} / \mathrm{s}^{2}$.

## Validate

The units for displacement are metres and metres per second for acceleration, which are correct. Both displacement and acceleration are positive, as they should be.

## 5. Frame the Problem

- Let time zero be the moment that Michael begins to accelerate.
- At time zero, Michael is 75 m behind Robert and will thus must run 75 m further than Robert in order to catch up with him.
- When Michael catches up to Robert, they will have run for the same amount of time.
- Michael is travelling with uniform acceleration. Thus, the equation of motion that relates displacement, initial velocity, acceleration, and time interval describes Michael's motion.
- Robert travels with constant velocity or uniform motion. Robert's motion can therefore be described by using the equation that defines velocity.


## Identify the Goal

Length of time it will take Michael to catch up with Robert in the race

## Variables and Constants

## Known

$a_{\mathrm{M}}=0.15 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$
$v_{\mathrm{M}_{(\mathrm{i})}}=3.8 \frac{\mathrm{~m}}{\mathrm{~s}}$
$v_{\mathrm{R}}=4.2 \frac{\mathrm{~m}}{\mathrm{~s}}$

## Unknown

$\Delta t$
$\Delta d_{M}$
$\Delta d_{R}+75 \mathrm{~m}$

## Strategy

Write a mathematical equation that states that the distance Michael runs is equal to the distance Robert runs during the time interval, plus the 75 m Michael has to make up.
Substitute the equation that defines the velocity for Robert for $\Delta d_{R}$. Substitute the equation of motion that relates displacement, initial velocity, acceleration, and the time interval for Michael for $\Delta d_{\mathrm{M}}$. The time interval from time zero is the same for the two runners when Michael catches up with Robert. Solve for $\Delta t$, the unknown, after substituting the known values into the equations.

## Calculations

$$
\Delta d_{\mathrm{M}}=\Delta d_{\mathrm{R}}+75 \mathrm{~m}
$$

$$
v_{\mathrm{M}} \Delta t+\frac{1}{2} a_{\mathrm{M}} \Delta t^{2}=75 \mathrm{~m}+v_{\mathrm{R}} \Delta t
$$

$$
\begin{aligned}
& 3.8 \frac{\mathrm{~m}}{\mathrm{~s}} \Delta t+\frac{1}{2}\left(0.15 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right) \Delta t^{2} \\
& =75 \mathrm{~m}+4.2 \frac{\mathrm{~m}}{\mathrm{~s}} \Delta t \\
& 3.8 \frac{\mathrm{~m}}{\mathrm{~s}} \Delta t+\left(0.075 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right) \Delta t^{2} \\
& =75 \mathrm{~m}+4.2 \frac{\mathrm{~m}}{\mathrm{~s}} \Delta t \\
& 3.8 \frac{\mathrm{~m}}{\mathrm{~s}} \Delta t-4.2 \frac{\mathrm{~m}}{\mathrm{~s}} \Delta t \\
& +\left(0.075 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right) \Delta t^{2}-75=0 \\
& -0.4 \frac{\mathrm{~m}}{\mathrm{~s}} \Delta t+\left(0.075 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right) \Delta t^{2}-75=0 \\
& \left(0.075 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right) \Delta t^{2}-0.4 \frac{\mathrm{~m}}{\mathrm{~s}} \Delta t-75=0
\end{aligned}
$$

Use the quadratic formula to solve for $\Delta t$, since it cannot be easily factored.

$$
\begin{aligned}
\Delta t & =\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \\
\Delta t & =\frac{-(-0.4) \pm \sqrt{(-0.4)^{2}-4(0.075)(-75)}}{2(0.075)} \\
\Delta t & =\frac{0.4 \pm \sqrt{0.16+22.5}}{0.15} \\
\Delta t & =\frac{0.4 \pm \sqrt{22.66}}{0.15} \\
& =\frac{0.4 \pm \sqrt{4.76}}{0.15}
\end{aligned}
$$

Exclude -29.07, since a negative time has no meaning in this situation.

$$
\begin{aligned}
\Delta t & =\frac{0.4+4.76}{0.15} & \Delta t & =\frac{0.4-4.76}{0.15} \\
\Delta t & =\frac{5.16}{0.15} & \Delta t & =\frac{-4.36}{0.15} \\
\therefore \Delta t & =34.4 \mathrm{~s} & \therefore \Delta t & =-29.07 \mathrm{~s}
\end{aligned}
$$

It will take Michael 34 s to catch Robert.

## Validate

The time is positive, and it seems to be reasonable.

## 6. Frame the Problem

- Sketch and label the situation.

- Since the acceleration of the race car is constant, the equation of motion that relates displacement, initial velocity, acceleration, and time interval describes its motion.


## Identify the Goal

How far the car has travelled (its displacement) after 8 s

## Variables and Constants

| Known | Unknown |
| :--- | :--- |
| $v_{\mathrm{i}}=200 \frac{\mathrm{~km}}{\mathrm{~h}}$ | $\Delta d$ |
| $\Delta t=8.0 \mathrm{~s}$ |  |
| $a=5.0 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}$ |  |

## Strategy

First, convert $200 \mathrm{~km} / \mathrm{h}$ to $\mathrm{m} / \mathrm{s}$, since acceleration and time are given in these units.
Select the equation of motion that relates the unknown variable, $\Delta d$, to the three known variables, $a, v_{\mathrm{i}}$, and $\Delta t$.
All of the needed quantities are known, so substitute them into the equation.

## Calculations

$\frac{200 \mathrm{kmt}}{\mathrm{h}} \times \frac{1 \mathrm{~h}}{3600 \mathrm{~s}} \times \frac{1000 \mathrm{~m}}{1 \mathrm{kmt}}=55.6 \frac{\mathrm{~m}}{\mathrm{~s}}$

$$
\Delta d=v_{\mathrm{i}} \Delta t+\frac{1}{2} a \Delta t^{2}
$$

$$
\begin{aligned}
\Delta d=55.6 & \frac{\mathrm{~m}}{\mathrm{~s}}(8.0 \mathrm{~s}) \\
& +\frac{1}{2}\left(5.0 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(8.0 \mathrm{~s})^{2}
\end{aligned}
$$

Simplify.

$$
\begin{aligned}
\Delta d & =444.8 \mathrm{~m}+160 \mathrm{~m} \\
\Delta d & =604.8 \mathrm{~m}=605 \mathrm{~m} \\
\Delta d & =604.8 \mathrm{~m}=600 \mathrm{~m}
\end{aligned}
$$

The race car travelled $6.0 \times 10^{2} \mathrm{~m}$ during the 8.0 s time interval.
Validate
The units cancelled to give metres for displacement, which is correct, and the displacement seems to be reasonable.

## 7. Frame the Problem

- Sketch and label the diagram of motion.

- The motorist must slow down and cover the 150 m in 10 s without stopping, and continue travelling when the light turns green.
- Since the acceleration of the car is constant, the equation of motion that relates to acceleration, displacement, initial velocity, and time interval describes the motion.


## Identify the Goal

(a) The acceleration of the car
(b) The speed of the car just as it passes the green light

## Variables and Constants

## Known

$v_{\mathrm{i}}=20.0 \frac{\mathrm{~m}}{\mathrm{~s}}$

## Unknown

$\Delta d=1.50 \times 10^{2} \mathrm{~m}$
$v_{\mathrm{f}}$
$\Delta t=10.0 \mathrm{~s}$

## Strategy

Select the equation of motion that relate the unknown variable, acceleration, to the three known variables, time, initial velocity, and displacement. Substitute and simplify.

## Calculations

Rearrange $\Delta d=v_{\mathrm{i}} \Delta t+\frac{1}{2} a \Delta t$
to solve for $a$ :

$$
\begin{aligned}
& a=\frac{2 \Delta d}{\Delta t^{2}}-\frac{2 v_{\mathrm{i}}}{\Delta t} \\
& a=\frac{2(150 \mathrm{~m})}{(10 \mathrm{~s})^{2}}-\frac{2(20) \frac{\mathrm{m}}{s}}{10 \mathrm{~s}} \\
& a=\frac{300 \mathrm{~m}}{100 \mathrm{~s}^{2}}-\frac{40 \mathrm{~m}}{10 \mathrm{~s}} \\
& a=30 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}-40 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \\
& a=-10 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}
\end{aligned}
$$

(a) The motorist's acceleration is $-10.0 \mathrm{~m} / \mathrm{s}^{2}$

Select the equation that relates final $\quad v_{\mathrm{f}}=v_{\mathrm{i}}+a \Delta t$ velocity to the initial velocity, acceleration, and time interval.

All of the needed quantities are known, so substitute them into the equation.

Simplify.
(b) The final velocity of the car just as it passes the green light is $10.0 \mathrm{~m} / \mathrm{s}$.

## Validate

The negative value of the acceleration indicated the motorist slowed down to a reasonable final velocity of $10 \mathrm{~m} / \mathrm{s}$.

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## 8. Frame the Problem

- Make a scale diagram of the problem. Choose a scale of $1.0 \mathrm{~cm}: 1.0 \times 10^{2} \mathrm{~km}$.
- The airplane's trip consisted of two separate displacements.
- The vector sum of the two displacement vectors yields one resultant vector that gives the airplane's final displacement.


## Identify the Goal

(a) The displacement, $\Delta \vec{d}_{\mathrm{R}}$, of the first two legs of the airplane's trip
(b) The [direction] the airplane needs in order to fly straight back

Variables and Constants
Known
$\Delta \vec{d}_{\mathrm{A}}=6.18 \times 10^{2} \mathrm{~km}\left[\mathrm{~N} 58.0^{\circ} \mathrm{W}\right]$
$\vec{d}_{B}=361 \mathrm{~km}\left[\mathrm{E} 35^{\circ} \mathrm{S}\right]$

## Strategy

Measure the length of the resultant vector in the scale diagram.

Multiply the length of the vector by the scale factor.

With a protractor, measure the angle between the horizontal axis and the resultant vector.
(a) The resultant displacement was $2.6 \times 10^{2} \mathrm{~km}\left[\mathrm{~W} 28^{\circ} \mathrm{N}\right]$.
(b) To return directly back to Sydney, the airplane must fly in a direction [E28 $\left.{ }^{\circ} \mathrm{S}\right]$.

## Validate

The total distance travelled was $979 \mathrm{~km}(618 \mathrm{~km}+361 \mathrm{~km})$. However, the airplane's path was not straight. The two legs of the trip, plus the final return leg, form a triangle.
Any side of a triangle must be shorter than the sum of the other two sides. Because the second leg of the trip is directed partially toward the starting point, you would expect the resultant vector to be much less than the sum of the other two sides. In fact, it is the shortest side.

## 9. Frame the Problem

- Make a scale diagram of the problem.

- The canoeist's trip consisted of two separate steps represented by the two displacement vectors in the diagram.
- The vector sum of the two vectors yields one resultant vector that shows the canoeist's final displacement.
- To travel straight home the canoeist will have to travel a displacement that is equal in magnitude to her resultant vector and opposite in direction.


## Identify the Goal

(a) The displacement $\Delta \vec{d}_{\mathrm{R}}$, of the two parts of the canoeist's trip
(b) The direction she would have to head her canoe to paddle straight home

## Variables and Constants

| Known | Unknown |
| :--- | :--- |
| $\Delta \vec{d}_{\mathrm{A}}=3.0 \mathrm{~km}[\mathrm{~N}]$ | $\Delta \vec{d}_{\mathrm{R}}$ |
| $\Delta \vec{d}_{\mathrm{B}}=4.0 \mathrm{~km}[\mathrm{~W}]$ | [direction] |

## Strategy

Measure the length of the resultant displacement vector in the scale diagram.
Multiply the length of the vector by the scale factor.

With a protractor, measure the angle

Calculations $\Delta \vec{d}_{\mathrm{R}}=2.5 \mathrm{~cm}$
$\Delta \vec{d}_{\mathrm{R}}=2.5 \mathrm{~cm}\left(\frac{2.0 \mathrm{~km}}{1.0 \mathrm{~cm}}\right)$
$\Delta \vec{d}_{\mathrm{R}}=5.0 \mathrm{~km}$
$\theta=37^{\circ}$
between the horizontal axis and the resultant vector.
(a) The resultant displacement is $5.0 \mathrm{~km}\left[\mathrm{~W} 37^{\circ} \mathrm{N}\right]$.
(b) The direction she would have to head her canoe to travel straight home is opposite to $\left[\mathrm{W} 37^{\circ} \mathrm{N}\right]$, or $\left[\mathrm{E} 37^{\circ} \mathrm{S}\right]$.

## Validate

The total distance the canoeist paddled was $7.0 \mathrm{~km}(3.0 \mathrm{~km}+4.0 \mathrm{~km})$. However, her path was not straight. The path the canoe travelled formed a triangle. Since any side of a triangle must be shorter than the sum of the other two, you would expect her trip will be shorter than 7.0 km . In fact it was 2.0 km shorter.

## 10. Frame the Problem

- Make a scale diagram of the problem. Choose a scale of $1.0 \mathrm{~cm}: 1.0 \mathrm{~km}$.
- The hiker's trip consisted of two separate displacements.
- The vector sum of the two displacement vectors yields one resultant vector that gives the hiker's final displacement.


## Identify the Goal

The displacement, $\Delta \vec{d}_{\mathrm{R}}$, of the first two legs of the hiker's trip.

## Variables and Constants

Known
Unknown
$\Delta \vec{d}_{\mathrm{A}}=4.5 \mathrm{~km}$
$\Delta \vec{d}_{\mathrm{R}}$
$\Delta \vec{d}_{\mathrm{B}}=6.4 \mathrm{~km}$
$\theta=60.0^{\circ}$

## Strategy

Measure the length of the resultant vector in the scale diagram.
Multiply the length of the vector by the scale factor.

With a protractor, measure the angle between the horizontal axis and the resultant vector.

The resultant displacement was 5.8 km [ $18^{\circ}$ away from the horizontal from the lookout].

## Validate

The total distance travelled was $10.9 \mathrm{~km}(4.5 \mathrm{~km}+6.4 \mathrm{~km})$. However, the hiker's path was not straight. The two legs of the trip, plus the final return leg form a triangle.
Any side of a triangle must be shorter than the sum of the other two sides. Because the second leg of the trip is directed partially toward the starting point (i.e. $60^{\circ}$ away), you expect the resultant vector to be much less than the sum of the other two sides. In fact, it is between the longest and shortest side.

## 11. Frame the Problem

- Make a scale diagram of the problem. Choose a scale of $1.0 \mathrm{~cm}: 6.0 \mathrm{~km}$.
- The boat's trip consisted of three separate displacements.
- The vector sum of the three displacement vectors yields one resultant vector that gives the boat's final displacement.


## Identify the Goal

(a) The displacement, $\Delta \vec{d}_{\mathrm{R}}$, of the three legs of the boat's trip
(b) The [direction] back to the boat's home port

Variables and Constants Known Unknown
$\Delta \vec{d}_{\mathrm{A}}=21.0 \mathrm{~km}[\mathrm{~N}]$ $\Delta \vec{d}_{\mathrm{R}}$
$\Delta \vec{d}_{\mathrm{B}}=31.0 \mathrm{~km}\left[\mathrm{~W} 30.0^{\circ} \mathrm{S}\right]$
[direction]
$\Delta \vec{d}_{\mathrm{C}}=36.0 \mathrm{~km}\left[\mathrm{~W} 10.0^{\circ} \mathrm{N}\right]$

## Strategy

Measure the length of the resultant vector in the scale diagram.

Calculations
$\Delta \vec{d}_{\mathrm{R}}=5.8 \mathrm{~cm}$

$$
\Delta \vec{d}_{\mathrm{R}}=5.8 \mathrm{~cm}
$$

$$
\Delta \vec{d}_{\mathrm{R}}=5.8 \mathrm{em}\left(\frac{1.0 \mathrm{~km}}{1.0 \mathrm{em}}\right)
$$ $\Delta \vec{d}_{\mathrm{R}}=5.8 \mathrm{em}\left(\frac{1.0 \mathrm{~km}}{1.0 \mathrm{em}}\right)$

$\Delta \vec{d}_{\mathrm{R}}=5.8 \mathrm{~km}$
$\theta=18^{\circ}$

$$
=18
$$

Multiply the length of the vector by the scale factor.
$\Delta \vec{d}_{\mathrm{R}}=10.5 \mathrm{em}\left(\frac{6.0 \mathrm{~km}}{1.0 \mathrm{em}}\right)$
$\Delta \vec{d}_{\mathrm{R}}=63.0 \mathrm{~km}$
$\theta=11.0^{\circ}$
With a protractor, measure the angle between the horizontal axis and the resultant vector.
(a) The resultant displacement was $63.0 \mathrm{~km}\left[\mathrm{~W} 11.0^{\circ} \mathrm{N}\right.$ ].
(b) To return directly back to the home port, the boat must travel in a direction [E11.0 ${ }^{\circ}$ ].

## Validate

The total distance the boat travelled was $87.0 \mathrm{~km}(21.0 \mathrm{~km}+30.0 \mathrm{~km}+36.0 \mathrm{~km})$. However, the boat's path was not straight. The first two segments of the trip make a triangle with the home port. But the third segment of the trip is in a direction further away from the home port. Therefore, you expect the resultant vector to be longer than any of the individual segments of the trip, though shorter than the total distance travelled. In fact, it is nearly twice as long as the longest segment.

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## 12. (a) Frame the Problem

- Establish a coordinate system and choose a scale. Use $1.0 \mathrm{~cm}: 2.0 \mathrm{~km}$.
- The first vector will be drawn from the origin and the negative of the second vector will be drawn from the tip of the first vector.
- The resultant vector will be drawn from the origin to the tip of the second vector.


## Identify the Goal

Three vectors: $\vec{P}-\vec{Q}, \vec{R}-\vec{Q}, \vec{Q}-\vec{R}$
Variables and Constants

| Known | Unknown |
| :--- | :--- |
| $\vec{P}=12 \mathrm{~km}[\mathrm{~N}]$ | $\vec{P}-\vec{Q}$ |
| $\vec{Q}=15 \mathrm{~km}[\mathrm{~S}]$ | $\vec{R}-\vec{Q}$ |
| $\vec{R}=10 \mathrm{~km}\left[\mathrm{~N} 30^{\circ} \mathrm{E}\right]$ | $\vec{Q}-\vec{R}$ |

## Strategy

(i) Measure the length of the resultant vector in the scale diagram.
Multiply the length of the vector by the scale factor.

Calculations
$\vec{P}-\vec{Q}=13.5 \mathrm{~cm}$
$\vec{P}-\vec{Q}=13.5 \mathrm{~mm}\left(\frac{2.0 \mathrm{~km}}{1.0 \mathrm{em}}\right)$
$\vec{P}-\vec{Q}=27 \mathrm{~km}$
$\theta=0^{\circ}$
between the horizontal axis and the resultant vector.
(ii) Repeat the above strategy for the next pair of vectors.

$$
\begin{aligned}
& \vec{R}-\vec{Q}=12.0 \mathrm{~cm} \\
& \vec{R}-\vec{Q}=12.0 \mathrm{em}\left(\frac{2.0 \mathrm{~km}}{1.0 \mathrm{~cm}}\right) \\
& \vec{R}-\vec{Q}=24 \mathrm{~km} \\
& \theta=12^{\circ}
\end{aligned}
$$

(iii) Repeat the above strategy for the last pair of vectors.

$$
\begin{aligned}
& \vec{Q}-\vec{R}=12.0 \mathrm{~cm} \\
& \vec{Q}-\vec{R}=12.0 \mathrm{em}\left(\frac{2.0 \mathrm{~km}}{1.0 \mathrm{em}}\right) \\
& \vec{Q}-\vec{R}=24 \mathrm{~km} \\
& \theta=12^{\circ}
\end{aligned}
$$

(a) The resultant vectors are: (i) $27 \mathrm{~km}[\mathrm{~N}]$ (ii) $24 \mathrm{~km}\left[12^{\circ} \mathrm{E}\right]$ and (iii) $24 \mathrm{~km}\left[\mathrm{~S} 12^{\circ} \mathrm{W}\right]$
(b) Frame the Problem

- Establish a coordinate system and choose a scale. Use $1.0 \mathrm{~cm}: 2.0 \mathrm{~km}$.
- Both the first and vectors will be drawn from the origin.
- The resultant vector will be drawn from the tip of the second vector to the tip of the first vector.


## Identify the Goal

Three vectors: $\vec{P}-\vec{Q}, \vec{R}-\vec{Q}, \vec{Q}-\vec{R}$
Variables and Constants

| Known | Unknown <br> $\vec{P}=12 \mathrm{~km}[\mathrm{~N}]$ |
| :--- | :--- |
| $\vec{P}=15 \mathrm{~km}[\mathrm{~S}]$ | $\vec{R}-\vec{Q}$ |
| $\vec{R}=10 \mathrm{~km}\left[\mathrm{~N} 30^{\circ} \mathrm{E}\right]$ | $\vec{P}-\vec{R}$ |

## Strategy

(i) Measure the length of the resultant vector in the scale diagram.

Multiply the length of the vector by the scale factor.

With a protractor, measure the angle between the horizontal axis and the resultant vector.
(ii) Repeat the above strategy for the next pair of vectors.
(iii) Repeat the above strategy for the last pair of vectors.

## Calculations

$\vec{P}-\vec{Q}=13.5 \mathrm{~cm}$
$\vec{P}-\vec{Q}=13.5 \mathrm{~m}\left(\frac{2.0 \mathrm{~km}}{1.0 \mathrm{~mm}}\right)$
$\vec{P}-\vec{Q}=27 \mathrm{~km}$
$\theta=0^{\circ}$
$\vec{R}-\vec{Q}=12.0 \mathrm{~cm}$
$\stackrel{\rightharpoonup}{R}-\stackrel{\rightharpoonup}{Q}=12.0 \mathrm{~m}\left(\frac{2.0 \mathrm{~km}}{1.0 \mathrm{em}}\right)$
$\vec{R}-\vec{Q}=24 \mathrm{~km}$
$\theta=12^{\circ}$
$\vec{P}-\vec{R}=3.0 \mathrm{~cm}$
$\vec{P}-\vec{R}=3.0 \mathrm{~mm}\left(\frac{2.0 \mathrm{~km}}{1.0 \mathrm{em}}\right)$
$\vec{P}-\vec{R}=6.0 \mathrm{~km}$
$\theta=34^{\circ}$

The resultant vectors are: (i) $27 \mathrm{~km}[\mathrm{~N}]$ (ii) $24 \mathrm{~km}\left[\mathrm{~N} 12^{\circ} \mathrm{E}\right]$ and (iii) $6.0 \mathrm{~km}\left[\mathrm{~W} 34^{\circ} \mathrm{N}\right]$

## Validate

The different methods give the same solutions for parts (i) and (ii). Particularly, for (a) (i), (ii) and (iii), since the vectors are pointed in opposite directions, you expect their difference to be larger than either of them, which it is. For (b)(iii), since the vectors are pointed in nearly the same direction, you expect their difference to be smaller than either of them, which it is.

## 13. Frame the Problem

- Make a scale diagram of the car's initial and final velocities. Use $1 \mathrm{~cm}: 5 \mathrm{~km}$.
- The car changes both its magnitude and direction of velocity.
- Either of the graphical vector subtraction methods can be used.


## Identify the Goal

The car's change in velocity, $\Delta \vec{v}$
Variables and Constants

| $\left.\begin{array}{ll}\text { Known } \\ \overrightarrow{v_{1}}=45 \frac{\mathrm{~km}}{\mathrm{~h}}\end{array} \mathrm{E}\right]$ | Unknown <br> $\overrightarrow{v_{2}}=50 \frac{\mathrm{~km}}{\mathrm{~h}}[\mathrm{~N}]$ |
| :--- | :--- |
| $\Delta \vec{v}$ |  |

## Strategy

Write the mathematical definition for the change in velocity

## Calculations

$\Delta \vec{v}=\overrightarrow{v_{2}}-\overrightarrow{v_{1}}$

Draw a coordinate system and choose
a scale. Use $1.0 \mathrm{~cm}=5 \mathrm{~km} / \mathrm{h}$
Put $\vec{v}_{2}$ in the coordinate system.
Place the tail of vector $-\vec{v}_{1}$ at the tip of $\vec{v}_{2}$ and draw $\Delta \vec{v}$.
Measure the magnitude and direction of $\Delta \vec{v}$.
Multiply the magnitude of the vector
by the scale factor $1.0 \mathrm{~cm}=5 \mathrm{~km} / \mathrm{h}$.

$$
\begin{aligned}
& |\Delta \stackrel{\rightharpoonup}{v}|=13.4 \mathrm{~cm}, \theta=48^{\circ} \\
& |\Delta \stackrel{\rightharpoonup}{v}|=13.4 \mathrm{~cm}\left(\frac{5 \frac{\mathrm{~km}}{\mathrm{~h}}}{1.0 \mathrm{em}}\right) \\
& |\Delta \vec{v}|=67 \frac{\mathrm{~km}}{\mathrm{~h}}
\end{aligned}
$$

The car's change in velocity was $67 \mathrm{~km} / \mathrm{h}\left[\mathrm{W} 48^{\circ} \mathrm{N}\right]$

## Validate

The magnitude of the change in the car's velocity is larger than the car's initial or final velocity, which it should be because the initial and final velocities are $90^{\circ}$ away from one another. In this case, the change in velocity makes up the hypotenuse of a right triangle, and the hypotenuse is the longest side of a right triangle. The solution can be checked using the other graphical vector subtraction method.

## 14. Frame the Problem

- Make a scale diagram of the airplane's initial and final velocities. Use $1 \mathrm{~cm}: 20 \mathrm{~km} / \mathrm{h}$.
- The airplane changes both its magnitude and direction of velocity.
- Either of the graphical vector subtraction methods can be used.


## Identify the Goal

The airplane's change in velocity, $\Delta \vec{v}$.
Variables and Constants

## Known

$\overrightarrow{v_{1}}=2.00 \times 10^{2} \frac{\mathrm{~km}}{\mathrm{~h}}\left[\mathrm{~S} 30.0^{\circ} \mathrm{W}\right]$
$\vec{v}_{2}=2.00 \times 10^{2} \frac{\mathrm{~km}}{\mathrm{~h}}[\mathrm{E}]$

## Strategy

Write the mathematical definition for the change in velocity

## Unknown

$\Delta \stackrel{\rightharpoonup}{v}$

## Calculations

$\Delta \vec{v}=\overrightarrow{v_{2}}-\overrightarrow{v_{1}}$

Draw a coordinate system and choose
a scale. Use $1.0 \mathrm{~cm}=20 \mathrm{~km} / \mathrm{h}$.
Put $\vec{v}_{2}$ in the coordinate system.
Place the tail of vector $-\overrightarrow{v_{1}}$ at the tip
of $\overrightarrow{v_{2}}$ and draw $\Delta \vec{v}$.
Measure the magnitude and direction of $\Delta \vec{v}$.
Multiply the magnitude of the vector
by the scale factor $1.0 \mathrm{~cm}=20 \mathrm{~km} / \mathrm{h}$.

$$
\begin{aligned}
& |\Delta \vec{v}|=17.3 \mathrm{~cm}, \theta=30^{\circ} \\
& |\Delta \stackrel{\rightharpoonup}{v}|=17.3 \mathrm{~m}\left(\frac{20 \frac{\mathrm{~km}}{\mathrm{~h}}}{1.0 \mathrm{em}}\right) \\
& |\Delta \vec{v}|=346 \frac{\mathrm{~km}}{\mathrm{~h}}
\end{aligned}
$$

The airplane's change in velocity was $346 \mathrm{~km} / \mathrm{h}\left[\mathrm{E} 30.0^{\circ} \mathrm{N}\right]$
Validate
The magnitude of the change in the airplane's velocity is larger than the airplane's initial or final velocity, which it should be because the initial and final velocities are more than $90^{\circ}$ away from one another. The solution can be checked using the other graphical vector subtraction method.

## 15. Frame the Problem

- Make a scale diagram of the puck's initial and final velocities. Use $1.0 \mathrm{~cm}: 2 \mathrm{~m} / \mathrm{s}$.
- The puck changes both its magnitude and direction of velocity.
- Either of the graphical vector subtraction methods can be used.


## Identify the Goal

The puck's change in velocity, $\Delta \stackrel{\rightharpoonup}{v}$.
Variables and Constants

| Known | Unknown |
| :--- | :--- |
| $\overrightarrow{v_{1}}=12 \frac{\mathrm{~m}}{\mathrm{~s}}\left[30^{\circ}\right.$ to the boards $]$ | $\Delta \vec{v}$ |
| $\overrightarrow{v_{2}}=10 \frac{\mathrm{~m}}{\mathrm{~s}}\left[25^{\circ}\right.$ to the boards $]$ |  |

## Strategy

Write the mathematical definition for the change in velocity
Draw a coordinate system and choose a scale. Use $1.0 \mathrm{~cm}=2 \mathrm{~m} / \mathrm{s}$.
Put $\overrightarrow{v_{2}}$ in the coordinate system.
Place the tail of vector $-\overrightarrow{v_{1}}$ at the tip
of $\overrightarrow{v_{2}}$ and draw $\Delta \vec{v}$.
Measure the magnitude and direction of $\Delta \stackrel{\rightharpoonup}{v}$.
Multiply the magnitude of the vector
by the scale factor $1.0 \mathrm{~cm}=2 \mathrm{~m} / \mathrm{s}$.

## Calculations

$\Delta \vec{v}=\overrightarrow{v_{2}}-\overrightarrow{v_{1}}$

The puck's change in velocity was $10 \mathrm{~m} / \mathrm{s}\left[7^{\circ}\right]$ away from the normal to the boards, towards the puck's initial direction.

## Validate

The magnitude of the change in the puck's velocity is between the puck's initial and final velocity. This is reasonable because you don't expect the puck to lose much energy as it deflects off the boards. The solution can be checked using the other graphical vector subtraction method.

## 16. Frame the Problem

- Make a scale diagram of the runner's velocity at the four points along the route. Use $1.0 \mathrm{~cm}: 1.0 \mathrm{~m} / \mathrm{s}$
- The runner changes both his magnitude and direction of velocity at each point along the route.
- Either of the graphical vector subtraction methods can be used.


## Identify the Goal

The runner's change in velocity, $\Delta \vec{v}_{21}, \Delta \overrightarrow{v_{31}}, \Delta \vec{v}_{41} \mathrm{t}$ three points along the route

## Variables and Constants

| Known | Unknown |
| :--- | :--- |
| $\overrightarrow{v_{1}}=3.5 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~S}]$ | $\Delta \overrightarrow{v_{21}}$ |
| $\overrightarrow{v_{2}}=5.0 \frac{\mathrm{~m}}{\mathrm{~s}}\left[\mathrm{~N} 12^{\circ} \mathrm{W}\right]$ | $\Delta \overrightarrow{v_{31}}$ |
| $\overrightarrow{v_{3}}=4.2 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~W}]$ | $\Delta \overrightarrow{v_{41}}$ |
| $\overrightarrow{v_{4}}=2.0 \frac{\mathrm{~m}}{\mathrm{~s}}\left[\mathrm{~S} 76^{\circ} \mathrm{E}\right]$ |  |

## Strategy

Write the mathematical definition for

## Calculations

 the change in velocity$$
\Delta \vec{v}=\overrightarrow{v_{2}}-\overrightarrow{v_{1}}
$$

Draw a coordinate system and choose a scale. Use $1.0 \mathrm{~cm}=1.0 \mathrm{~m} / \mathrm{s}$.
Put $\vec{v}_{2}$ in the coordinate system.
Place the tail of vector $-\overrightarrow{v_{1}}$ at the tip of $\vec{v}_{2}$ and draw $\Delta \vec{v}_{21}$.
Measure the magnitude and direction

$$
\begin{aligned}
& \left|\Delta \vec{v}_{21}\right|=8.4 \mathrm{~cm}, \theta=7^{\circ} \\
& \left|\Delta \overrightarrow{v_{21}}\right|=8.4 \mathrm{em}\left(\frac{1.0 \frac{\mathrm{~m}}{\mathrm{~s}}}{1.0 \mathrm{em}}\right) \\
& \left|\Delta \overrightarrow{v_{21}}\right|=8.4 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& \left|\Delta \overrightarrow{v_{31}}\right|=5.5 \mathrm{~cm}, \theta=40^{\circ} \\
& \left|\Delta \overrightarrow{v_{31}}\right|=5.5 \mathrm{em}\left(\frac{1.0 \frac{\mathrm{~m}}{\mathrm{~s}}}{1.0 \mathrm{~mm}}\right) \\
& \left|\Delta \overrightarrow{v_{31}}\right|=5.5 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& \left|\Delta \overrightarrow{v_{41}}\right|=3.6 \mathrm{~cm}, \theta=57^{\circ} \\
& \left|\Delta \overrightarrow{v_{41}}\right|=3.6 \mathrm{em}\left(\frac{1.0 \frac{\mathrm{~m}}{\mathrm{~s}}}{1.0 \mathrm{em}}\right) \\
& \left|\Delta \vec{v}_{41}\right|=3.6 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

Repeat the above procedure for $\Delta \vec{v}_{41}$.
(a) The runner's change in velocity was $8.4 \mathrm{~m} / \mathrm{s}\left[\mathrm{N} 7^{\circ} \mathrm{W}\right]$
(b) The runner's change in velocity was $5.5 \mathrm{~m} / \mathrm{s}\left[\mathrm{N} 40^{\circ} \mathrm{E}\right]$
(c) The runner's change in velocity was $3.6 \mathrm{~m} / \mathrm{s}\left[\mathrm{E} 57^{\circ} \mathrm{N}\right]$

## Validate

(a) The individual velocities are in opposite directions, so the magnitude of their difference should be larger than either of them, which it is.
(b) The change in velocity forms the hypotenuse of a right triangle, which is larger than either of the sides of the triangle, as is calculated.
(c) The change in velocity must be less than the sum of the two sides of the triangle, which it is.

## Practice Problem Solutions

## Student Textbook pages 102-103

## 17. Frame the Problem

- Make a diagram of the problem.
- The hang-glider changes course while flying; displacement is a vector that depends on the initial and final positions.
- Velocity is the vector quotient of displacement and the time interval.
- The direction of the hang-glider's average velocity is therefore the same as the direction of her displacement.


## Identify the Goal

The average velocity of the trip, $\vec{v}_{\text {ave }}$.
Variables and Constants

| Known | Unknown |
| :--- | :--- |
| $\Delta \vec{d}_{\mathrm{A}}=6.0 \mathrm{~km}[\mathrm{~S}]$ | $\Delta \vec{d}_{\mathrm{R}}$ |
| $\Delta \vec{d}_{\mathrm{B}}=4.0 \mathrm{~km}[\mathrm{NW}]$ | $\overrightarrow{\mathrm{v}}_{\text {ave }}$ |

$\Delta t=45 \mathrm{~min}=0.75 \mathrm{~h}$

## Strategy

## Calculations

Choose a coordinate system; use a
scale of $1 \mathrm{~cm}: 1.0 \mathrm{~km}$.
Draw the first displacement vector.
Draw the second displacement vector from the tip of the first displacement vector. Measure the length and direction of the resultant vector.
Multiply the length of the resultant vector by the scale factor.

Use the equation that defines average velocity to calculate the magnitude of the average velocity

$$
\begin{aligned}
& \left|\Delta \vec{d}_{\mathrm{R}}\right|=4.3 \mathrm{~cm}, \theta=43^{\circ} \\
& \left|\Delta \overrightarrow{\Delta d}_{\mathrm{R}}\right|=4.3 \mathrm{~mm}\left(\frac{1.0 \mathrm{~km}}{1.0 \mathrm{em}}\right) \\
& \left|\Delta \vec{d}_{\mathrm{R}}\right|=4.3 \mathrm{~km} \\
& \vec{v}_{\text {ave }}=\frac{\Delta \vec{d}}{\Delta t} \\
& \vec{v}_{\text {ave }}=\frac{4.3 \mathrm{~km}}{0.75 \mathrm{~h}}=5.7 \mathrm{~km} / \mathrm{h}
\end{aligned}
$$

The average velocity for the trip was $5.7 \mathrm{~km} / \mathrm{h}\left[\mathrm{S} 42^{\circ} \mathrm{W}\right]$

## Validate

Because you don't know the times for the individual segments of the trip, you have nothing to which to compare the average velocity. The wind caused a significant change of course. Since the hang-glider made a sharp turn, you expect the total displacement, 4.3 km , to be shorter than the sum of the two segments of the trip $(6.0 \mathrm{~km}+4.0 \mathrm{~km})$, which it is.

## 18. Frame the Problem

- Make a diagram of the problem.
- The plane's flight is in two segments which are given as displacements; displacement is a vector that depends on the initial and final positions.
- Velocity is the vector quotient of displacement and the time interval.
- The direction of the plane's average velocity is the same as the direction of its displacement.


## Identify the Goal

(a) The average velocity of the trip, $\vec{v}_{\text {ave }}$.
(b) The velocity needed to return in $1.0 \mathrm{~h}, \overrightarrow{v_{\mathrm{R}}}$

Variables and Constants

Known
$\Delta \vec{d}_{\mathrm{A}}=195 \mathrm{~km}\left[\mathrm{~N} 15^{\circ} \mathrm{W}\right]$
Unknown
$\Delta \vec{d}_{\mathrm{B}}=149 \mathrm{~km}\left[\mathrm{~N} 33^{\circ} \mathrm{E}\right]$
$\Delta t=3.75 \mathrm{~h}$
$\Delta \vec{d}_{\mathrm{R}}$
$\vec{v}_{\text {ave }}$
$\bar{v}_{\mathrm{R}}$

## Strategy

## Calculations

Choose a coordinate system; use a scale of $1 \mathrm{~cm}: 10.0 \mathrm{~km}$.
Draw the first displacement vector.
Draw the second displacement vector from the tip of the first displacement vector. Measure the length and direction of the resultant vector.
Multiply the length of the resultant vector by the scale factor.
$\left|\Delta \vec{d}_{\mathrm{R}}\right|=31.5 \mathrm{~cm} \theta=84^{\circ}$
$\left|\Delta d_{\mathrm{R}}\right|=31.5 \mathrm{em}\left(\frac{10.0 \mathrm{~km}}{1.0 \mathrm{em}}\right)$
$\left|\Delta d_{\mathrm{R}}\right|=315 \mathrm{~km}$
Use the equation that defines average velocity to calculate the magnitude of the average velocity
To find the velocity required to return in 1.0 h , use the equation for velocity.

$$
\vec{v}_{\text {ave }}=\frac{\Delta \bar{d}}{\Delta t}
$$

$$
\vec{v}_{\text {ave }}=\frac{315 \mathrm{~km}}{3.75 \mathrm{~h}}=84 \mathrm{~km} / \mathrm{h}
$$

$$
\bar{v}_{\mathrm{R}}=\frac{\Delta \bar{d}}{\Delta t}
$$

$$
\vec{v}_{\mathrm{R}}=\frac{315 \mathrm{~km}}{1.0 \mathrm{~h}}=315 \mathrm{~km} / \mathrm{h}
$$

(a) The average velocity for the trip was $84 \mathrm{~km} / \mathrm{h}\left[\mathrm{E} 84^{\circ} \mathrm{N}\right]$.
(b) To fly back in 1.0 h , the pilot needs to fly at $3.2 \times 10^{2} \mathrm{~km} / \mathrm{h}\left[\mathrm{W} 84^{\circ} \mathrm{S}\right]$

## Validate

The average velocity is small for a plane, but considering the distances travelled in the rather long period of time, the value is reasonable.

## 19. Frame the Problem

- Make a diagram of the problem.
- The canoeist's trip is in two segments which are given in terms of velocities and time intervals.
- The total displacement is the vector sum of the two displacement vectors.


## Identify the Goal

The displacement of the canoeist, $\Delta \vec{d}_{\text {total }}$
Variables and Constants
Known
Unknown

$$
\begin{aligned}
& \Delta t_{\mathrm{A}}=30.0 \mathrm{~min} \\
& \Delta t_{\mathrm{B}}=15.0 \mathrm{~min} \\
& \vec{v}_{\text {A ave }}=3.0 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~N}] \\
& \overrightarrow{v_{\mathrm{B}}}{ }^{\text {ave }}=2.5 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~W}]
\end{aligned}
$$

$$
\Delta \vec{d}_{\mathrm{A}}
$$

## Strategy

Use the velocity for segment A to calculate the displacement for segment A.
Multiply by the number of seconds in 1 minute to convert minutes to seconds.

Calculations
$\Delta \vec{d}_{\mathrm{A}}=\vec{v}_{\mathrm{A}} \Delta t_{\mathrm{A}}$

$$
\begin{aligned}
& \Delta \vec{d}_{\mathrm{A}}=3.0 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~N}](30.0 \mathrm{mint})\left(\frac{60 \mathrm{~s}}{1 \mathrm{minti}}\right) \\
& \Delta \vec{d}_{\mathrm{A}}=5400 \mathrm{~m}[\mathrm{~N}]
\end{aligned}
$$

Choose a coordinate system; use a scale of 1 cm : 1000 m .
Draw the displacement vector, $\Delta \vec{d}_{\mathrm{A}}$ with its tail at the origin.
Use the velocity for segment B to calculate the displacement for segment B.
Multiply by the number of seconds in 1 minute to convert minutes to seconds.
Draw the displacement vector, $\Delta \vec{d}_{\mathrm{B}}$, with its tail at the tip of vector $A$.
Draw the resultant displacement vector,
$\Delta \vec{d}_{\text {total }}$, from the tail of A to the tip of B.
Measure the magnitude and direction of the total displacement vector.
Multiply the magnitude of the vector by the scale factor.

$$
\begin{aligned}
& \left|\Delta \vec{d}_{\text {total }}\right|=5.8 \mathrm{~cm}, \theta=23^{\circ} \\
& \left|\Delta \vec{d}_{\text {total }}\right|=5.8 \mathrm{em}\left(\frac{1000 \mathrm{~m}}{1.0 \mathrm{em}}\right) \\
& \left|\Delta \vec{d}_{\text {total }}\right|=5800 \mathrm{~m}
\end{aligned}
$$

The displacement for the canoe trip is $5.8 \times 10^{3} \mathrm{~m}\left[\mathrm{~N} 23^{\circ} \mathrm{W}\right]$.

## Validate

In this case, the total displacement is the hypotenuse of a right triangle. The hypotenuse is longer than either of the two individual sides of the triangle, as is the case here.

## 20. Frame the Problem

- Make a diagram of the problem.
- The hiker's trip is in three segments which are given in terms of velocities and time intervals.
- The total displacement is the vector sum of the three displacement vectors.


## Identify the Goal

(a) The displacement of the hiker, $\Delta \vec{d}_{\text {total }}$
(b) The hiker's average velocity, $\vec{v}_{\text {ave }}$

## Variables and Constants

## Known

$\Delta t_{\mathrm{A}}=48.0 \mathrm{~min}$
$\Delta t_{\mathrm{B}}=40.0 \mathrm{~min}$
$\Delta t_{\mathrm{C}}=1.5 \mathrm{~h}$
${\overrightarrow{v_{\mathrm{A}}}}$ ave $=5.0 \frac{\mathrm{~km}}{\mathrm{~h}}\left[\mathrm{~N} 35^{\circ} \mathrm{E}\right]$
${\overrightarrow{v_{\mathrm{B}}}}$ ave $=4.5 \frac{\mathrm{~km}}{\mathrm{~h}}[\mathrm{~W}]$
$\Delta \vec{d}_{\mathrm{C}}=6.0 \mathrm{~km}\left[\mathrm{~N} 30^{\circ} \mathrm{W}\right]$

## Unknown

$\Delta \vec{d}_{\mathrm{A}}$
$\Delta \vec{d}_{\mathrm{B}}$
$\Delta \vec{d}_{\mathrm{C}}$
$\Delta \vec{d}_{\text {total }}$
$\overrightarrow{v a v e}^{\text {ave }}$

## Strategy

Use the velocity for segment A to calculate the displacement for segment A.
Multiply by the number of minutes in 1 hour to convert minutes to hours.

Choose a coordinate system; use a scale of $1 \mathrm{~cm}: 1 \mathrm{~km}$.
Draw the displacement vector, $\Delta \vec{d}_{\mathrm{A}}$ with its tail at the origin.
Use the velocity for segment B to calculate the displacement for segment B.
Multiply by the number of minutes in 1 hour to convert minutes to hours.
Draw the displacement vector, $\Delta \vec{d}_{\mathrm{B}}$, with its tail at the tip of vector A.
Draw the displacement vector C, with its tail at the tip of vector $B$.
Draw the resultant displacement vector, $\Delta \vec{d}_{\text {total }}$, from the tail of A to the tip of C.
Measure the magnitude and
direction of the total displacement vector.

Multiply the magnitude of the vector by the scale factor.
Use the equation that defines average velocity to find the average velocity.
Convert all the times to hours.

$$
\begin{aligned}
& \vec{v}_{\text {ave }}=\frac{9.2 \mathrm{~km}}{48 \mathrm{mminf}\left(\frac{1 \mathrm{~h}}{60 \mathrm{~min}}\right)+40 \mathrm{mminf( }\left(\frac{1 \mathrm{~h}}{60 \text { min }}\right)+1.5 \mathrm{~h}} \\
& \vec{v}_{\text {ave }}=3.1 \mathrm{~km} / \mathrm{h}
\end{aligned}
$$

## Calculations <br> $\Delta \vec{d}_{\mathrm{A}}=\vec{v}_{\mathrm{A}} \Delta t_{\mathrm{A}}$

$\Delta \vec{d}_{\mathrm{A}}=5.0 \frac{\mathrm{~km}}{\mathrm{H}}\left[\mathrm{N} 35^{\circ} \mathrm{E}\right](48.0 \mathrm{mintr})\left(\frac{1 \mathrm{t}}{60 \mathrm{mmint}}\right)$
$\Delta \vec{d}_{\mathrm{A}}=4.0 \mathrm{~km}\left[\mathrm{~N} 35^{\circ} \mathrm{E}\right]$

$$
\Delta \vec{d}_{\mathrm{B}}=\vec{v}_{\mathrm{B}} \Delta t_{\mathrm{B}}
$$

$$
\Delta \vec{d}_{\mathrm{B}}=4.5 \frac{\mathrm{~km}}{\mathrm{tr}}[\mathrm{~W}](40 \mathrm{mintr})\left(\frac{1 \mathrm{t}}{60 \mathrm{mintin}}\right)
$$

$$
\Delta \vec{d}_{\mathrm{B}}=3.0 \mathrm{~km}[\mathrm{~W}]
$$

$$
\begin{aligned}
& \left|\Delta \stackrel{\rightharpoonup}{d}_{\text {total }}\right|=9.2 \mathrm{~cm}, \theta=24^{\circ} \\
& \left|\Delta \stackrel{\rightharpoonup}{d}_{\text {total }}\right|=9.2 \mathrm{em}\left(\frac{1.0 \mathrm{~km}}{1.0 \mathrm{em}}\right) \\
& \left|\Delta \stackrel{\rightharpoonup}{d}_{\text {total }}\right|=9.2 \mathrm{~km}
\end{aligned}
$$

(a) The displacement is $9.2 \mathrm{~km}\left[\mathrm{~N} 24^{\circ} \mathrm{W}\right]$.
(b) The average velocity is $3.1 \mathrm{~km} / \mathrm{h}\left[\mathrm{N} 24^{\circ} \mathrm{W}\right]$.

## Validate

On the third segment of the trip, the hiker travelled further away from the starting point than on the other segments, so the total displacement should be larger than any individual displacement, which it is. Note that average velocity is different from average speed, so it cannot be determined from the speeds on the individual segments. For the average velocity, a magnitude of $3.1 \mathrm{~km} / \mathrm{h}$ seems reasonable considering the data.

## Practice Problem Solutions

## Student Textbook page 110

## 21. Frame the Problem

- Make a diagram of the problem.
- The kayak is moving relative to the river and the river is moving relative to the shore.
- The velocity of the kayak relative to the shore is the sum of the velocity of the kayak relative to the river and the velocity of the river relative to the shore.
- The vectors for the velocity of the kayak relative to the river and the river relative to the shore are in opposite directions.


## Identify the Goal

The velocity of the river relative to the shore, $\vec{\nu}_{\text {rs }}$.
Variables and Constants

| Known | wn |
| :---: | :---: |
| $\vec{\nu}_{\text {kr }}=3.5 \mathrm{~m} / \mathrm{s}$ [upstream] | $\vec{\nu}_{\text {rs }}$ |
| $\overrightarrow{v_{\mathrm{ks}}}=1.7 \mathrm{~m} / \mathrm{s}$ [upstream] |  |

## Strategy

Write an equation that describes the situation.
Substitute known values.
Solve for the unknown, $\vec{v}_{\text {rs }}$

## Calculations

$$
\vec{v}_{\mathrm{ks}}=\vec{v}_{\mathrm{kr}}+\vec{v}_{\mathrm{rs}}
$$

$$
1.7 \frac{\mathrm{~m}}{\mathrm{~s}}=3.5 \frac{\mathrm{~m}}{\mathrm{~s}}+{\overrightarrow{v_{\mathrm{rs}}}}
$$

$$
\vec{\nu}_{\mathrm{rs}}=1.7 \frac{\mathrm{~m}}{\mathrm{~s}}-3.5 \frac{\mathrm{~m}}{\mathrm{~s}}
$$

$$
\vec{v}_{\mathrm{rs}}=-1.8 \frac{\mathrm{~m}}{\mathrm{~s}}
$$

The velocity of the current is $1.8 \mathrm{~m} / \mathrm{s}$ [downstream].
Validate
Because the kayak and the river are travelling opposite directions, they must have opposite signs, which they do. Observers on the shore see the kayak paddling about half as fast as he thinks he's going. The other half of the speed must be due to the river.

## 22. Frame the Problem

- Make a diagram of the problem.
- The jet-ski is heading due south, but the river is carrying it to the west.
- The velocity of the jet-ski relative to the shore, $\vec{v}_{\text {js }}$. will be the hypotenuse of a right triangle whose other sides are the velocity of the river relative to the shore, $\vec{\nu}_{\text {rs }}$ and the velocity of the jet-ski relative to the river, $\vec{v}_{\mathrm{jr}}$.


## Identify the Goal

The velocity of the jet-ski relative to the shore, $\vec{v}_{\mathrm{js}}$
Variables and Constants

## Known

$\vec{v}_{\mathrm{j} \mathrm{r}}=11 \mathrm{~m} / \mathrm{s}[\mathrm{S}]$
$\vec{v}_{\mathrm{rs}}=5.0 \mathrm{~m} / \mathrm{s}[\mathrm{W}]$

Unknown
$\stackrel{\rightharpoonup}{v_{j}}$

## Strategy

Use the Pythagorean theorem to find the magnitude of the velocity of the jet-ski relative to the shore.

Find the direction of this velocity from the tangent.

## Calculations

$$
\begin{aligned}
\left|\vec{v}_{\mathrm{js}}\right|^{2} & =\left|\vec{v}_{\mathrm{j}}\right|^{2}+\left|\vec{v}_{\mathrm{rs}}\right|^{2} \\
\left|\vec{v}_{\mathrm{js}}\right| & =\sqrt{\left(11 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}+\left(5.0 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}} \\
\left|\vec{v}_{\mathrm{js}}\right| & =12.1 \frac{\mathrm{~m}}{\mathrm{~s}} \\
\tan \theta & =\frac{\mid v_{\mathrm{s}}}{\left|\mathrm{r}_{\mathrm{r} \mid}\right|}=\frac{5.0}{11.0}=0.4545 \\
\theta & =\tan ^{-1} 0.4545 \\
\theta & =24.4^{\circ}
\end{aligned}
$$

The velocity of the jet-ski relative to the shore is $12 \mathrm{~m} / \mathrm{s}\left[\mathrm{S} 24^{\circ} \mathrm{W}\right]$.

## Validate

Because the jet-ski is being carried by the current in the river, its velocity relative to the shore should be larger than its velocity relative to the river and the velocity of the river relative to the shore, which it is.

## 23. Frame the Problem

- Make a sketch of the problem.

- The pilot will be carried east by the wind, so she will aim her plane slightly west of the lake.
- Her ground speed, $\vec{\nu}_{\mathrm{pg}}$,will be greater than her air speed ( $\vec{\nu}_{\mathrm{pw}}=210 \mathrm{~km} / \mathrm{h}$ ), because, in a sense, she's going with the wind ( $\vec{\nu}_{\mathrm{wg}}=40 \mathrm{~km} / \mathrm{h}$ )


## Identify the Goal

(a) The direction that the pilot should head her plane, $\theta$
(b) The velocity of the plane relative to the ground, $\vec{v}_{\mathrm{pg}}$, for that direction
(c) The time it will take her to reach the lake, $\Delta t$

Variables and Constants

| Known | Unknown |
| :--- | :--- |
| $\vec{v}_{\mathrm{pw}}=210 \mathrm{~km} / \mathrm{h}$ | $\vec{v}_{\mathrm{pg}}$ |
| $\vec{v}_{\mathrm{wg}}=40 \mathrm{~km} / \mathrm{h}[\mathrm{E}]$ | [direction, $\theta]$ |

$\Delta t$

## Strategy

## Calculations

As the vectors do not form a right triangle, the sine law can be used to solve for the unknown direction, $\theta$.

$$
\begin{aligned}
& \frac{\left|\vec{v}_{\mathrm{pw}}\right|}{\sin 60}=\frac{\left|\vec{v}_{\mathrm{wg}}\right|}{\sin \theta} \\
& \sin \theta=\frac{\left|\vec{v}_{\mathrm{wg}}\right| \sin 60}{\left|\vec{v}_{\mathrm{pw}}\right|} \\
& \sin \theta=\frac{40 \frac{\mathrm{~km}}{\mathrm{~h}} \sin 60}{210 \frac{\mathrm{~km}}{\mathrm{~h}}} \\
& \sin \theta=0.16496 \\
& \theta=\sin ^{-1} 0.16496 \\
& \theta=9.49^{\circ} \\
& \varphi=30-\theta \\
& \varphi=30-9.49 \\
& \varphi=20.51^{\circ} \\
& \frac{\left|\vec{v}_{\mathrm{pw}}\right|}{\sin 60}=\frac{\left|\vec{v}_{\mathrm{pg}}\right|}{\sin \alpha} \\
&\left|\vec{v}_{\mathrm{pg}}\right|=\frac{\left|\vec{v}_{\mathrm{pw}}\right| \sin \alpha}{\sin 60} \\
&\left|\vec{v}_{\mathrm{pg}}\right|=\frac{210 \frac{\mathrm{~km}}{\mathrm{~h}} \sin (180-60-9.49)}{\sin 60} \\
&\left|\vec{v}_{\mathrm{pg}}\right|=227.1 \frac{\mathrm{~km}}{\mathrm{~h}} \\
& \vec{v}_{\mathrm{ave}}=\frac{\Delta \vec{d}}{\Delta t} \\
& \Delta t=\frac{\Delta \vec{d}}{\vec{v}_{\mathrm{ave}}} \\
& \Delta t=\frac{250 \mathrm{kmm}}{227.1 \frac{\mathrm{~km}}{\mathrm{~h}}} \\
& \Delta t=1.10 \mathrm{~h}
\end{aligned}
$$

To get the heading from North, note that $\theta+\varphi=30^{\circ}$
the velocity of the plane relative to the

$$
\text { ground, }{\stackrel{\rightharpoonup}{v_{\mathrm{pg}}}}
$$

The angle, $\alpha$ can be determined from e other two angles in the triangle,
$\alpha+\theta+60=180^{\circ}$
(a) The direction she should head her plane is $\left[\mathrm{N} 20.5^{\circ} \mathrm{E}\right]$.
(b) With this heading, her velocity relative to the ground will be $227 \mathrm{~km} / \mathrm{h}\left[\mathrm{N} 30.0^{\circ} \mathrm{E}\right]$.
(c) It will take her 1.10 h to reach her destination.

## Validate

The wind speed is not that strong compared to her velocity with respect to the air so you don't expect her heading to be too far away from her destination. In fact, she only has to point her plane $9.5^{\circ}$ away from her destination, (or $20.5^{\circ}$ away from North), which seems reasonable. Her velocity with respect to the ground is larger than her velocity with respect to the air, as it should be.
The units cancelled in the time determination to give the result in hours.

## 24. Frame the Problem

- Make a diagram of the problem.
- The airplane's trip is in three segments which are given in terms of displacements and time intervals.
- The total displacement is the vector sum of the three displacement vectors.
- Velocity is the vector quotient of the displacement and the time interval.


## Identify the Goal

(a) The displacement of the airplane, $\Delta \vec{d}_{\text {total }}$
(b) The velocities, $\vec{v}_{\mathrm{A}_{\text {ave }}}, \vec{v}_{\mathrm{B}_{\text {ave }}}, \vec{v}_{\mathrm{C}_{\text {ave }}}$, for each segment of the trip.
(c) The average velocity for the total trip, $\vec{v}_{\text {ave }}$.

## Variables and Constants

Known
Unknown

| $\Delta t_{\mathrm{A}}=20.0 \mathrm{~min}$ | $\Delta \vec{d}_{\text {total }}$ |
| :--- | :--- |
| $\Delta t_{\mathrm{B}}=40.0 \mathrm{~min}$ | $\vec{v}_{\text {A ave }}$ |
| $\Delta t_{\mathrm{C}}=12.0 \mathrm{~min}$ | $\vec{v}_{\mathrm{B}}$ ave |
| $\Delta \vec{d}_{\mathrm{A}}=1.0 \times 10^{2} \mathrm{~km}[\mathrm{~N}]$ | $\vec{v}_{\mathrm{C}}$ ave |
| $\Delta \vec{d}_{\mathrm{B}}=1.5 \times 10^{2} \mathrm{~km}[\mathrm{~W}]$ | $\vec{v}_{\text {ave }}$ |
| $\Delta \vec{d}_{\mathrm{C}}=5.0 \times 10^{2} \mathrm{~km}[\mathrm{~S}]$ |  |

## Strategy

Choose a coordinate system; use a
scale of $1 \mathrm{~cm}: 50 \mathrm{~km}$.
Draw the displacement vector, $\Delta \vec{d}_{\mathrm{A}}$ with its tail at the origin.
Draw the displacement vector, $\Delta \vec{d}_{\mathrm{B}}$, with its tail at the tip of vector A . Draw the displacement vector $C$, with its tail at the tip of vector B. Draw the resultant displacement vector, $\Delta \vec{d}_{\text {total }}$, from the tail of A to the tip of C.
Measure the magnitude and direction of the total displacement vector.
Multiply the magnitude of the vector by the scale factor.

Calculate the velocity for each segment from the displacement and the time interval.
and the time interval.
Convert the time interval to hours. $\quad \vec{v}_{\mathrm{A}}=\frac{\Delta \vec{d}_{\mathrm{A}}}{\Delta t_{\mathrm{A}}}$

Use the equation that defines average velocity to find the average velocity.
Convert all the times to hours.

## Calculations

$$
\left|\Delta \stackrel{\rightharpoonup}{d}_{\text {total }}\right|=3.2 \mathrm{~cm}, \theta=18^{\circ}
$$

$$
\left|\Delta \stackrel{\rightharpoonup}{d}_{\text {total }}\right|=3.2 \mathrm{~m}\left(\frac{50 \mathrm{~km}}{1.0 \mathrm{~m}}\right)
$$

$$
\left|\Delta \vec{d}_{\text {total }}\right|=160 \mathrm{~km}
$$

$$
\overrightarrow{v_{\mathrm{A}}}=\frac{\Delta \vec{d}_{\mathrm{A}}}{\Delta t_{\mathrm{A}}}
$$

$$
\overrightarrow{v_{\mathrm{A}}}=\frac{\Delta \vec{d}_{\mathrm{A}}}{\Delta t_{\mathrm{A}}}
$$

$$
\overrightarrow{v_{\mathrm{A}}}=\frac{1.0 \times 10^{2} \mathrm{~km}[\mathrm{~N}]}{20.0 \mathrm{mmin}\left(\frac{1}{60 \mathrm{~h} \operatorname{mint}}\right)}
$$

$$
\overrightarrow{v_{\mathrm{A}}}=3.0 \times 10^{2} \mathrm{~km}[\mathrm{~N}]
$$

$$
\stackrel{\rightharpoonup}{v_{\mathrm{B}}}=\frac{\Delta \vec{d}_{\mathrm{B}}}{\Delta t_{\mathrm{B}}}
$$

$$
\overrightarrow{v_{\mathrm{B}}}=\frac{1.5 \times 10^{2} \mathrm{~km}[\mathrm{~W}]}{40.0 \mathrm{minin}\left(\frac{1 \mathrm{~h}}{60 \min }\right)}
$$

$$
\stackrel{\rightharpoonup}{v_{\mathrm{B}}}=2.25 \times 10^{2} \mathrm{~km}[\mathrm{~W}]
$$

$$
\vec{v}_{\mathrm{C}}=\frac{\Delta \vec{d}_{\mathrm{C}}}{\Delta t_{\mathrm{C}}}
$$

$$
\stackrel{\rightharpoonup}{v_{\mathrm{C}}}=\frac{5.0 \times 10^{1} \mathrm{~km}[\mathrm{~S}]}{12.0 \min \left(\frac{1 \mathrm{~h}}{60 \min }\right)}
$$

$$
\vec{v}_{\mathrm{C}}=2.50 \times 10^{2} \mathrm{~km}[\mathrm{~S}]
$$

$$
\vec{v}_{\mathrm{ave}}=\frac{\Delta \vec{d}}{\Delta t}
$$

$$
\begin{aligned}
& \vec{v}_{\text {ave }}=\frac{160 \mathrm{~km}}{20.0 \operatorname{minin}\left(\frac{1 \mathrm{~h}}{60 \min }\right)+40.0 \operatorname{minin}\left(\frac{1 \mathrm{~h}}{60 \min }\right)+12.0 \min } \\
& \vec{\imath} \quad-122221 \mathrm{~m} / \mathrm{h}
\end{aligned}
$$

$$
\stackrel{\rightharpoonup}{v a v e}=133.33 \mathrm{~km} / \mathrm{h}
$$

(a) The displacement is $1.6 \times 10^{2} \mathrm{~km}\left[\mathrm{~W} 18^{\circ} \mathrm{N}\right]$.
(b) The velocities for the three segments are: $3.0 \times 10^{2} \mathrm{~km} / \mathrm{h}[\mathrm{N}], 2.2 \times 10^{2} \mathrm{~km} / \mathrm{h}[\mathrm{W}], 2.5 \times 10^{2} \mathrm{~km} / \mathrm{h}[\mathrm{S}]$
(c) The average velocity for the total trip is: $1.3 \times 10^{2} \mathrm{~km} / \mathrm{h}$.

## Validate

The displacement, 160 km , is less than the total distance travelled
$(100 \mathrm{~km}+150 \mathrm{~km}+50 \mathrm{~km})$, as it should be. The individual velocities seem reasonable. The pilot went both north and south, so you expect the average velocity for the trip to be less than the individual velocities for those segments, which it is. The total displacement can be checked using the other graphical method.

## 25. Frame the Problem

- Make a diagram of the problem.
- The swimmer must swim at an angle upstream so that between his efforts and the river, he travels straight across.
- The velocity of the swimmer relative to the river, $\vec{v}_{\text {sr }}$ will be the hypotenuse of a right triangle whose other sides are the velocity of the river relative to the ground (or the shore), $\vec{\nu}_{\mathrm{rg}}$, and the velocity of the swimmer relative to the ground, $\vec{v}_{\mathrm{sg}}$.
- The velocity of the river relative to the ground, $\vec{\nu}_{\mathrm{rg}}$, can be determined from the distance and time interval that the stick travels.
- The velocity of the swimmer relative to the ground, $\vec{v}_{\text {sg }}$, needs to be determined before the time to cross can be determined.


## Identify the Goal

(a) The direction the swimmer should head, $\theta$, in order to land directly across.
(b) The time it will take the swimmer to cross the river, $\Delta t$.

Variables and Constants

| Known | Unknown |
| :--- | :--- |
| $\left\|\vec{v}_{\text {sr }}\right\|=1.9 \mathrm{~m} / \mathrm{s}$ | $\theta$ |
| $\Delta \vec{d}_{\text {river }}=120 \mathrm{~m}[\mathrm{~N}]$ | $\vec{v}_{\text {rg }}$ |
| $\Delta \vec{d}_{\text {stick }}=24 \mathrm{~m}[\mathrm{~W}]$ | $\vec{v}_{\text {sg }}$ |
| $\Delta t_{\text {stick }}=30.0 \mathrm{~s}$ | $\Delta t$ |

## Strategy

Use the definition of velocity to find the velocity of the current from the data for the stick.

Now, the magnitudes of the hypotenuse, $\vec{v}_{\mathrm{sr}}$, and opposite side of the triangle, $\vec{\nu}_{\mathrm{rg}}$, are known. Find the angle, $\theta$.

## Calculations

$$
\begin{aligned}
& \vec{v}_{\text {rg }}=\frac{\Delta \vec{d}_{\text {sidk }}}{\Delta \Delta_{\text {tick }}} \\
& \stackrel{\rightharpoonup}{\mathrm{v}}_{\mathrm{rg}}=\frac{24 \mathrm{~m}[\mathrm{~W}]}{30.0 \mathrm{~s}} \\
& \vec{\nu}_{\text {rg }}=0.8 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~W}] \\
& \sin \theta=\frac{\left|\vec{v}_{\mathrm{g}}\right|}{\left|\overrightarrow{v s r s}_{\mathrm{s}}\right|} \\
& \sin \theta=\frac{0.8 \frac{\pi}{s}}{1.9 \frac{\pi}{9}} \\
& \sin \theta=0.42105 \\
& \theta=\sin ^{-1} 0.42105 \\
& \theta=24.9^{\circ}
\end{aligned}
$$

Use the Pythagorean theorem to find the magnitude of the velocity of the swimmer relative to the ground, $\stackrel{\rightharpoonup}{v_{s g}}$

$$
\begin{aligned}
\left|\vec{v}_{\mathrm{sr}}\right|^{2} & =\left|\vec{v}_{\mathrm{sg}}\right|^{2}+\left|\vec{v}_{\mathrm{rg}}\right|^{2} \\
\left|\vec{v}_{\mathrm{sg}}\right|^{2} & =\left|\vec{v}_{\mathrm{sr}}\right|^{2}-\left|\vec{v}_{\mathrm{rg}}\right|^{2} \\
\left|\vec{v}_{\mathrm{sg}}\right| & =\sqrt{\left(1.9 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}-\left(0.8 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}} \\
\left|\vec{v}_{\mathrm{sg}}\right| & =1.723 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

Find the time to cross the river from the

$$
\begin{aligned}
& \vec{v}_{\mathrm{sg}}=\frac{\Delta \vec{d}_{\text {river }}}{\Delta t} \\
& \Delta t=\frac{\Delta \vec{d}_{\text {river }}}{\vec{v}_{\mathrm{sg}}} \\
& \Delta t=\frac{120 \mathrm{~m}[\mathrm{~A}]}{1.723 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~A}]} \\
& \Delta t=69.63 \mathrm{~s}
\end{aligned}
$$

(a) The swimmer should head in a direction $\left[\mathrm{N} 25^{\circ} \mathrm{E}\right]$ to arrive directly across the river.
(b) The trip would take 69 seconds.

## Validate

In each case, the units cancelled to give the correct units for the desired quantity. You expect the swimmer's velocity relative to the ground to be less than the swimmer's velocity relative to the river because he has to direct himself upstream. These values were observed.

## 26. Frame the Problem

- Make a diagram of the problem.
- The hiker's trip is in three segments which are given in terms of displacements.
- The total displacement is the vector sum of the three displacement vectors.
- Speed is the scalar quotient of the distance and the time interval. It can be used to find the total time of the trip.


## Identify the Goal

(a) The displacement of the hiker, $\Delta \vec{d}_{\text {total }}$.
(b) The direction, $\theta$, the hiker would have to head to return to her starting point.
(c) The total time of the trip.

Variables and Constants

| Known | Unknown |
| :--- | :--- |
| $\Delta \vec{d}_{\mathrm{A}}=4.0 \mathrm{~km}\left[\mathrm{~N} 40.0^{\circ} \mathrm{W}\right]$ | $\Delta \vec{d}_{\text {total }}$ |
| $\Delta \vec{d}_{\mathrm{B}}=3.0 \mathrm{~km}\left[\mathrm{E} 10.0^{\circ} \mathrm{N}\right]$ | $\theta$ |
| $\Delta \vec{d}_{\mathrm{C}}=2.5 \mathrm{~km}\left[\mathrm{~S} 40.0^{\circ} \mathrm{W}\right]$ | $\Delta t$ |
| $v=4.0 \mathrm{~km} / \mathrm{h}$ |  |

## Strategy

## Calculations

Choose a coordinate system; use a scale of $1 \mathrm{~cm}: 1.0 \mathrm{~km}$
Draw the displacement vector, $\Delta \vec{d}_{\mathrm{A}}$ with its tail at the origin.
Draw the displacement vector, $\Delta \vec{d}_{\mathrm{B}}$, with its tail at the tip of vector $A$.
Draw the displacement vector $C$, with its tail at the tip of vector $B$.
Draw the resultant displacement vector,
$\Delta \vec{d}_{\text {total }}$, from the tail of A to the tip of C.

Measure the magnitude and direction of
the total displacement vector.

$$
\begin{aligned}
& \left|\Delta \vec{d}_{\text {total }}\right|=2.1 \mathrm{~cm}, \theta=54^{\circ} \\
& \left|\Delta \vec{d}_{\text {total }}\right|=2.1 \mathrm{em}\left(\frac{1.0 \mathrm{~km}}{1.0 \mathrm{em}}\right) \\
& \left|\Delta \vec{d}_{\text {total }}\right|=2.1 \mathrm{~km} \\
& v=\frac{d}{\Delta t} \\
& \Delta t=\frac{d}{v} \\
& \Delta t=\frac{(4.0+3.0+2.5) \mathrm{km}}{4.0 \frac{\mathrm{~km}}{\mathrm{~h}}} \\
& \Delta t=2.375 \mathrm{~h}
\end{aligned}
$$

Multiply the magnitude of the vector by the scale factor.
(a) The displacement is $2.1 \mathrm{~km}\left[\mathrm{~W} 54^{\circ} \mathrm{N}\right]$.
(b) To head straight back, she should walk in the direction $\left[\mathrm{S} 54^{\circ} \mathrm{E}\right]$.
(c) The total time taken for the trip is 2.4 h .

Validate
In each case, the units cancelled to give the correct units for the desired quantity. The displacement, 2.1 km , is less than the total distance travelled $(4.0 \mathrm{~km}+3.0 \mathrm{~km}+2.5 \mathrm{~km})$, as it should be. Given the speed that she walks, hiking about 10 km in just over 2 h seems reasonable.

## 27. Frame the Problem

- Make a diagram of the problem.
- The canoeist is heading due east, but the river is carrying the canoe to the south.
- The velocity of the canoeist relative to the shore, $\vec{v}_{\mathrm{cs}}$ will be the hypotenuse of a right triangle whose other sides are the velocity of the river relative to the shore, $\vec{v}_{\mathrm{rS}}$, and the velocity of the canoeist relative to the river, $\vec{v}_{\mathrm{cr}}$.


## Identify the Goal

The velocity of the canoe relative to the shore, $\vec{v}_{\text {cs }}$
Variables and Constants

| Known | Unknown |
| :--- | :--- |
| $\overrightarrow{v_{\mathrm{cr}}}=1.5 \mathrm{~m} / \mathrm{s}[\mathrm{E}]$ | $\vec{v}_{\mathrm{cs}}$ |
| $\overrightarrow{v_{\mathrm{rs}}}=0.50 \mathrm{~m} / \mathrm{s}[\mathrm{S}]$ |  |

## Strategy

Use the Pythagorean theorem to find the magnitude of the velocity of the canoe relative to the shore.

Find the direction of this velocity from the tangent.

$$
\begin{aligned}
& \text { Calculations } \\
& \begin{aligned}
\left|\vec{v}_{\mathrm{cs}}\right|^{2} & =\left|\vec{v}_{\mathrm{cr}}\right|^{2}+\left|\vec{v}_{\mathrm{rs}}\right|^{2} \\
\left|\vec{v}_{\mathrm{cs}}\right| & =\sqrt{\left(1.5 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}+\left(0.50 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}} \\
\left|\vec{v}_{\mathrm{cs}}\right| & =1.58 \frac{\mathrm{~m}}{\mathrm{~s}} \\
\tan \theta & =\frac{\left|v_{\mathrm{rs}}\right|}{\left|v_{\mathrm{j}}\right|}=\frac{0.50 \frac{\mathrm{~m}}{\mathrm{~s}}}{1.5 \frac{\mathrm{~m}}{\mathrm{~m}}}=0.3333 \\
\theta & =\tan ^{-1} 0.3333 \\
\theta & =18.4^{\circ}
\end{aligned}
\end{aligned}
$$

The velocity of the canoeist relative to the shore is $1.6 \mathrm{~m} / \mathrm{s}\left[\mathrm{E} 18^{\circ} \mathrm{S}\right]$.

## Validate

In each case, the units cancelled to give the correct units for the desired quantity. Because the canoeist is being carried by the current in the river, the canoe's velocity relative to the shore should be larger than its velocity relative to the river and the velocity of the river relative to the shore, which it is.

## Answers to Problems for Understanding

## Student Textbook pages 116-117

11. The initial velocity of the runner was $3 \mathrm{~m} / \mathrm{s}$.

$$
v_{\mathrm{f}}=v_{\mathrm{i}}+a \Delta t
$$

Solve for $v_{\mathrm{i}}$

$$
\begin{aligned}
v_{\mathrm{f}} & =v_{\mathrm{i}}+a \Delta t \\
& =6.4 \frac{\mathrm{~m}}{\mathrm{~s}}-\left(0.3 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(12 \mathrm{~s}) \\
& =2.8 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& =3 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

12. The baseball would have a velocity of $1.9 \mathrm{~m} / \mathrm{s}$ [down] after 4.0 s .

$$
\begin{aligned}
v_{\mathrm{f}} & =v_{\mathrm{i}}+a \Delta t \\
v_{\mathrm{f}} & =4.5 \frac{\mathrm{~m}}{\mathrm{~s}}+\left(-1.6 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(4.0 \mathrm{~s}) \\
& =4.5 \frac{\mathrm{~m}}{\mathrm{~s}}-6.4 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& =-1.9 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

13. (a) The final velocity of the car is $17 \mathrm{~m} / \mathrm{s}$.

$$
\begin{aligned}
\Delta d & =\left(\frac{v_{\mathrm{f}}+v_{\mathrm{f}}}{2}\right) \Delta t \\
50 \mathrm{~m} & =\left(\frac{0+v_{\mathrm{f}}}{2}\right)(6.0 \mathrm{~s}) \\
50 \mathrm{~m} & =\left(v_{\mathrm{f}}\right) 3.0 \mathrm{~s} \\
v_{\mathrm{f}} & =16.7 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

(b) The acceleration of the car is $2.8 \mathrm{~m} / \mathrm{s}^{2}$.

$$
\begin{aligned}
a & =\frac{v_{\mathrm{f}}-v_{\mathrm{i}}}{\Delta t} \\
& =\frac{16.7 \frac{\mathrm{~m}}{\mathrm{~s}}-0}{6.0 \mathrm{~s}} \\
& =2.8 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}
\end{aligned}
$$

14. (a) The cyclist travels 27 m during the 4.0 s interval.

$$
\begin{aligned}
\Delta d & =v_{\mathrm{i}} \Delta t+\frac{1}{2} a \Delta t^{2} \\
& =\left(5.6 \frac{\mathrm{~m}}{\mathrm{~s}}\right)(4.0 \mathrm{~s})+\frac{1}{2}\left(0.6 \frac{\mathrm{~m}}{\mathrm{~s}}\right)(4.0 \mathrm{~s})^{2} \\
& =22.4 \mathrm{~m}+4.8 \mathrm{~m} \\
& =27.2 \mathrm{~m}
\end{aligned}
$$

(b) The cyclist attains a velocity of $8.0 \mathrm{~m} / \mathrm{s}$.

$$
\begin{aligned}
v_{\mathrm{f}} & =v_{\mathrm{i}}+a \Delta t \\
& =5.6 \frac{\mathrm{~m}}{\mathrm{~s}}+0.6 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}(4.0 \mathrm{~s}) \\
& =5.6 \frac{\mathrm{~m}}{\mathrm{~s}}+2.4 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& =8.1 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

15. (a) The acceleration of the truck is $a=-1.2 \mathrm{~m} / \mathrm{s}^{2}$.

To derive an equation that relates initial and final velocities and distance to acceleration, start with the following.
$a=\frac{v_{\mathrm{f}}-v_{\mathrm{i}}}{\Delta t}$
Solve for $\Delta t$ and substitute it into the following.

$$
v_{\mathrm{f}}=v_{\mathrm{i}}+a \Delta t
$$

Rearrange the resulting equation to obtain the following (see Think It Through on page 82 of the textbook).

$$
\begin{aligned}
& v_{\mathrm{f}}^{2}=v_{1}^{2}+2 a d \\
& \left(14 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}=\left(22 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}+2 a(125 \mathrm{~m}) \\
& 196 \frac{\mathrm{~m}^{2}}{\mathrm{~s}^{2}}=484 \frac{\mathrm{~m}^{2}}{\mathrm{~s}^{2}}+a(250 \mathrm{~m}) \\
& a=\frac{(196-484) \frac{\mathrm{m}^{2}}{\mathrm{~s}^{2}}}{250 \mathrm{~m}} \\
& a=-1.152 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}
\end{aligned}
$$

(b) It took 6.9 s for the truck driver to change his speed.

Solve for $\Delta t$ in the following.

$$
\begin{aligned}
a & =\frac{v_{\mathrm{f}}-v_{\mathrm{i}}}{\Delta t} \\
\Delta t & =\frac{v_{\mathrm{f}}-v_{\mathrm{i}}}{a} \\
\Delta t & =\frac{14 \frac{\mathrm{~m}}{\mathrm{~s}}-22 \frac{\mathrm{~m}}{\mathrm{~s}}}{-1.152 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}} \\
& =6.94 \mathrm{~s}
\end{aligned}
$$

16. Find the initial velocity.

$$
\begin{aligned}
& v_{\mathrm{f}}=v_{\mathrm{i}}+a \Delta t \\
& v_{\mathrm{i}}=v_{\mathrm{f}}-a \Delta t \\
& v_{\mathrm{i}}=\left(-30 \frac{\mathrm{~m}}{\mathrm{~s}}\right)-\left(-3.2 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right) 8.0 \mathrm{~s} \\
& v_{\mathrm{i}}=\left(-30 \frac{\mathrm{~m}}{\mathrm{~s}}\right)+25.6 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& v_{\mathrm{i}}=-4.4 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

Find the displacement.

$$
\begin{aligned}
\Delta d & =v_{\mathrm{i}} \Delta t+\frac{1}{2} a \Delta t^{2} \\
& =\Delta d\left(-4.4 \frac{\mathrm{~m}}{\mathrm{~s}}\right) 8.0 \mathrm{~s}+\frac{1}{2}\left(-3.2 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right)(8.0 \mathrm{~s})^{2} \\
& =\Delta d(-4.4 \mathrm{~m})-102.4 \mathrm{~m} \\
& =\Delta d=-106.8 \mathrm{~m}
\end{aligned}
$$

The sky diver's displacement was $-1.1 \times 10^{2} \mathrm{~m}$ or $1.1 \times 10^{2} \mathrm{~m}$ [down].
17. When the police car catches up with the speeder, both vehicles will have travelled the same distance during the same time interval.
(a) It will take the police car 23 s to catch up to the speeder.

$$
\begin{aligned}
& \Delta d_{\text {speeder }}=\Delta d_{\text {police }} \\
& v \Delta t=\frac{1}{2} a \Delta t^{2} \\
& 24 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \Delta t=\frac{1}{2}\left(2.1 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right) \Delta t^{2} \\
& \frac{1}{2}\left(2.1 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right) \Delta t^{2}-24 \frac{\mathrm{~m}}{\mathrm{~s}} \Delta t=0 \\
& 1.05 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \Delta t^{2}-24 \frac{\mathrm{~m}}{\mathrm{~s}} \Delta t=0 \\
& \left(1.05 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \Delta t-24 \frac{\mathrm{~m}}{\mathrm{~s}}\right) \Delta t=0 \\
& \Delta t=0-1.05 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \Delta t-24 \frac{\mathrm{~m}}{\mathrm{~s}}=0 \\
& 1.05 \frac{\mathrm{~m}}{\mathrm{~s}^{2}} \Delta t=24 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& \Delta t=22.86 \mathrm{~s}
\end{aligned}
$$

Note that the solution, $\Delta t=0$, represents the fact that the time at which the speeder passed the police car the first time. $\Delta t=23 \mathrm{~s}$ is the second time that the speeder and the police car were at the same position, that is, when the police car caught up with the speeder.
(b) Each vehicle will travel $5.50 \times 10^{2} \mathrm{~m}$. Use either formula $\Delta d=v \Delta t$ for the speeder or $\Delta d=\frac{1}{2} a \Delta t^{2}$ for the police car to obtain the distance.

$$
\begin{aligned}
\Delta d & =v \Delta t \\
\Delta d & =24 \frac{\mathrm{~m}}{\mathrm{~s}}(22.86 \mathrm{~s}) \\
& =548.57 \mathrm{~m}
\end{aligned}
$$

18. (a) Refer to the vector diagram and calculations below.

| Scale $1.0 \mathrm{~cm}=20 \mathrm{~km} / \mathrm{h}$ |
| :--- |
|  |

$$
\begin{aligned}
&|\vec{v}|^{2}=v_{\mathrm{x}}^{2}+v_{\mathrm{y}}^{2} \\
&|\Delta \stackrel{\rightharpoonup}{v}|^{2}=\left(-50 \frac{\mathrm{~km}}{\mathrm{~h}}\right)^{2}+\left(50 \frac{\mathrm{~km}}{\mathrm{~h}}\right)^{2} \\
&|\Delta \stackrel{\rightharpoonup}{v}|^{2}=2500\left(\frac{\mathrm{~km}}{\mathrm{~h}}\right)^{2}+2500\left(\frac{\mathrm{~km}}{\mathrm{~h}}\right)^{2} \\
&|\Delta \vec{v}|^{2}=5000\left(\frac{\mathrm{~km}}{\mathrm{~h}}\right)^{2} \\
&|\Delta \vec{v}|=71 \frac{\mathrm{~km}}{\mathrm{~h}} \\
& \tan \theta=\frac{-50 \frac{\mathrm{~km}}{-5 \frac{\mathrm{k}}{\mathrm{k}}}=+1}{\theta}=\tan ^{-1} 1=45^{\circ} \\
& \Delta \vec{v}=71 \frac{\mathrm{~km}}{\mathrm{~h}}\left[\mathrm{~W} 45^{\circ} \mathrm{S}\right] \text { or } 71 \frac{\mathrm{~km}}{\mathrm{~h}}[\mathrm{SW}]
\end{aligned}
$$

(b) The acceleration during the turn is $3.9 \mathrm{~m} / \mathrm{s}^{2}[\mathrm{SW}]$, as shown.
$71 \frac{\mathrm{~km}}{\mathrm{~h}} \times \frac{1 \mathrm{~h}}{3600 \mathrm{~s}} \times \frac{1000 \mathrm{~m}}{1 \mathrm{~km}}=19.72 \frac{\mathrm{~m}}{\mathrm{~s}}$
$\vec{a}=\frac{\Delta \vec{v}}{\Delta t}$
$\vec{a}=\frac{\left.19.72 \frac{\mathrm{~m}}{\mathrm{~s}} \mathrm{SW}\right]}{5.0 \mathrm{~s}}$
$\vec{a}=3.9 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}[\mathrm{SW}]$
19. (a) Refer to the vector diagram below.

Scale $1.0 \mathrm{~cm}=1.0 \mathrm{~km}$

(b) The total displacement from the scale diagram above is $3.6 \mathrm{~km}\left[\mathrm{~S} 34^{\circ} \mathrm{W}\right]$.
20. Refer to the scale vector diagram and calculations below.

| Scale $1.0 \mathrm{~cm}=2.0 \mathrm{~km}$ |
| :--- |
| $\Delta \vec{d}=6.6 \mathrm{~km}\left[\mathrm{~N} 31^{\circ} \mathrm{W}\right]$ |
| $\|\vec{d}\|^{2}=d_{\mathrm{x}}^{2}+d_{\mathrm{y}}^{2}$ |
| $\|\Delta \vec{d}\|^{2}=(-3.4 \mathrm{~km})^{2}+(5.6 \mathrm{~km})^{2}$ |
| $\|\Delta \vec{d}\|^{2}=11.56 \mathrm{~km}^{2}+31.36 \mathrm{~km}{ }^{2}$ |
| $\|\Delta \vec{d}\|^{2}=42.92 \mathrm{~km}^{2}$ |
| $\|\Delta \vec{d}\|=6.6 \mathrm{~km}$ |
| $\tan \theta=\frac{-3.4 \mathrm{~km}}{+5.6 \mathrm{~km}=-0.607}$ |
| $\theta=\tan { }^{-1}-0.607=-31^{\circ}$ |
| $\Delta \vec{d}=6.6 \mathrm{~km}\left[\mathrm{~N} 31^{\circ} \mathrm{W}\right]$ |
| (b) $\Delta t_{\mathrm{total}}=1 \mathrm{~h}+0.5 \mathrm{~h}=1.5 \mathrm{~h}$ |
| $\vec{v}=\frac{\Delta \vec{d}}{\Delta t}$ |
| $\vec{v}=\frac{6.6 \mathrm{~km}\left[\mathrm{~N} 31^{\circ} \mathrm{W}\right]}{1.5 \mathrm{~h}}$ |
| $\vec{v}=4.4 \frac{\mathrm{~km}}{\mathrm{~h}}\left[\mathrm{~N} 31^{\circ} \mathrm{W}\right]$ |

21. (a) The scale vector diagram below represents the change in velocity

(b) The following is a calculation for the change in velocity.

$$
\begin{aligned}
& |\vec{v}|^{2}=v_{\mathrm{x}}^{2}+v_{\mathrm{y}}^{2} \\
& |\Delta \vec{v}|^{2}=\left(-5.6 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}+\left(5.6 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2} \\
& |\Delta \vec{v}|^{2}=31.36\left(\frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}+31.36\left(\frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2} \\
& |\Delta \vec{v}|^{2}=62.72\left(\frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2} \\
& |\Delta \vec{v}|=7.9 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& \tan \theta=\frac{-5.6 \frac{\mathrm{~m}}{\mathrm{~s}}}{+5.6 \frac{\mathrm{~m}}{\mathrm{~s}}}=-1 \\
& \theta \\
& \theta \tan ^{-1}-1=-45^{\circ} \\
& \Delta \vec{v}=7.9 \frac{\mathrm{~m}}{\mathrm{~s}}\left[\mathrm{~N} 45^{\circ} \mathrm{W}\right] \text { or } 7.9 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{NW}]
\end{aligned}
$$

22. (a) Refer to the scale vector diagram and the calculations below.


$$
\Delta \overrightarrow{d_{\mathrm{x}}}=\overrightarrow{v_{\mathrm{x}}} \times \Delta t
$$

$$
\Delta \vec{d}_{\mathrm{x}}=6.0 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~W}] \times 45 \mathrm{~min} \times \frac{60 \mathrm{~s}}{1 \mathrm{~min}}
$$

$$
\Delta \vec{d}_{\mathrm{x}}=16.2 \mathrm{~km}[\mathrm{~W}]
$$

$$
\Delta \vec{d}_{\mathrm{y}}=\overrightarrow{v_{\mathrm{y}}} \times \Delta t
$$

$$
\Delta{\overrightarrow{d_{y}}}_{y}=4.0 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~S}] \times 30 \mathrm{~min} \times \frac{60 \mathrm{~s}}{1 \mathrm{~min}}
$$

$$
\Delta \vec{d}_{\mathrm{x}}=7.2 \mathrm{~km}[\mathrm{~S}]
$$

$$
\begin{aligned}
|\Delta \vec{d}|^{2} & =\left|\vec{d}_{x}\right|^{2}+\left|\Delta \vec{d}_{y}\right|^{2} \\
|\Delta \vec{d}|^{2} & =(-16.2 \mathrm{~km})^{2}+(-7.2 \mathrm{~km})^{2} \\
|\Delta \vec{d}|^{2} & =262.4 \mathrm{~km}^{2}+51.8 \mathrm{~km}^{2} \\
|\Delta \vec{d}|^{2} & =314.28 \mathrm{~km}^{2} \\
|\Delta \vec{d}| & =17.7 \mathrm{~km}=18 \\
\tan \theta & =\frac{\Delta \vec{d}_{y}}{\Delta \vec{d}_{x}} \\
\tan \theta & =\frac{-7.2 \mathrm{~km}}{-16.2 \mathrm{~km}} \\
\theta & =\tan ^{-1} 0.444 \\
\theta & =24^{\circ} \\
\Delta \vec{d} & =18 \mathrm{~km}^{\circ}\left[\mathrm{W} 24^{\circ} \mathrm{S}\right]
\end{aligned}
$$

(b) $\Delta t_{\text {total }}=45 \mathrm{~min}+30 \mathrm{~min}=75 \mathrm{~min}$
$75 \mathrm{~min} \times \frac{1 \mathrm{~h}}{60 \mathrm{~min}}=1.25 \mathrm{~h}$

$$
\vec{v}=\frac{\Delta \vec{d}}{\Delta t}
$$

$$
\stackrel{\rightharpoonup}{v}=\frac{18 \mathrm{~km}\left[\mathrm{~W} 24^{\circ} \mathrm{S}\right]}{1.25 \mathrm{~h}}
$$

$$
\vec{v}=14.4 \frac{\mathrm{~km}}{\mathrm{~h}}\left[\mathrm{~W} 24^{\circ} \mathrm{S}\right]=14 \frac{\mathrm{~km}}{\mathrm{~h}}\left[\mathrm{~W} 24^{\circ} \mathrm{S}\right]
$$

23. (a) Refer to the scale vector diagram and calculations below.

$\vec{v}_{\mathrm{Tw}}=2.5 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~N}]$
$\vec{v}_{\mathrm{ws}}=1.2 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~S}]$
$\vec{v}_{\mathrm{Ts}}=\vec{v}_{\mathrm{Tw}}+\vec{v}_{\mathrm{ws}}$
$\vec{v}_{\mathrm{Ts}}=2.5 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~N}]+1.2 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~S}]$
$\vec{v}_{\mathrm{Ts}}=1.3 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~N}]$
(b) If Thao swims downstream, her velocity relative to the shore is $3.7 \mathrm{~m} / \mathrm{s}[\mathrm{S}]$ as shown in the above diagram and in the calculations below.

$$
\begin{aligned}
& \vec{v}_{\mathrm{Tw}}=2.5 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~S}] \\
& v_{\mathrm{ws}}=1.2 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~S}] \\
& \vec{v}_{\mathrm{Ts}}=\vec{v}_{\mathrm{Tw}}+\vec{v}_{\mathrm{ws}} \\
& \vec{v}_{\mathrm{Ts}}=2.5 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~S}]+1.2 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~S}] \\
& \vec{v}_{\mathrm{Ts}}=3.7 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~N}]
\end{aligned}
$$

24. (a) Refer to the scale vector diagram below.

$$
\begin{aligned}
\vec{v}_{\mathrm{ws}} & =\frac{4.2 \mathrm{~m}[\mathrm{~S}]}{5.0 \mathrm{~s}} \\
\vec{v}_{\mathrm{ws}} & =0.84 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~S}] \\
\left|\vec{v}_{\mathrm{P}}\right| & =1.9 \frac{\mathrm{~m}}{\mathrm{~s}} \\
\sin \theta & =\frac{0.84}{1.9} \\
\sin \theta & =0.4421 \\
\theta & =\sin ^{-1} 0.4421 \\
\theta & =26^{\circ}
\end{aligned}
$$

$\therefore$ His direction should be $\left[\mathrm{E} 26^{\circ} \mathrm{N}\right]$. He should swim at $1.9 \mathrm{~m} / \mathrm{s}\left[\mathrm{E} 26^{\circ} \mathrm{N}\right]$.
(b) $\quad\left|\vec{v}_{\mathrm{Pw}}\right|^{2}=\left|\vec{v}_{\mathrm{Ps}}\right|^{2}+\left|\vec{v}_{\mathrm{ws}}\right|^{2}$ $\left(1.9 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}=\left|{\overrightarrow{v_{\mathrm{Ps}}}}\right|^{2}+\left(0.84 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}$

$$
\left|\vec{v}_{\mathrm{Ps}}\right|^{2}=3.61\left(\frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}-0.7056\left(\frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}
$$

$$
\left|{\stackrel{\rightharpoonup}{v_{\mathrm{Ps}}}}\right|^{2}=2.9044\left(\frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}
$$

$$
\vec{v}_{\mathrm{P}_{\mathrm{s}}}=1.7 \frac{\mathrm{~m}}{\mathrm{~s}}
$$

$\therefore$ His velocity relative to the shore is $1.7 \mathrm{~m} / \mathrm{s}[\mathrm{E}]$.
(c) $\quad \Delta t=\frac{\Delta \vec{d}}{\overrightarrow{v_{\mathrm{Ps}}}}$

$$
\begin{aligned}
\Delta t & =\frac{4800 \mathrm{~m}}{1.7 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{E}]} \\
\Delta t & =2824 \mathrm{~s} \times \frac{1 \mathrm{~min}}{60 \mathrm{~s}}
\end{aligned}
$$

$$
\therefore \Delta t=47 \mathrm{~min}
$$

It will take the physics teacher 47 min to cross the lake.

25. Refer to the scale vector diagram and the calculations below.


$$
\begin{aligned}
v_{\mathrm{ws}(\mathrm{x})} & =\left|\vec{v}_{\mathrm{ws}}\right| \cos \theta_{\mathrm{ws}} \\
v_{\mathrm{ws}(\mathrm{x})} & =1.2 \frac{\mathrm{~m}}{\mathrm{~s}} \cos 70^{\circ} \\
v_{\mathrm{ws}(\mathrm{x})} & =1.2 \frac{\mathrm{~m}}{\mathrm{~s}}(0.3420) \\
v_{\mathrm{ws}(\mathrm{x})} & =0.410 \frac{\mathrm{~m}}{\mathrm{~s}} \\
v_{\mathrm{ws}(\mathrm{y})} & =\left|\vec{v}_{\mathrm{ws}}\right| \sin \theta_{\mathrm{ws}} \\
v_{\mathrm{ws}(\mathrm{y})} & =1.2 \frac{\mathrm{~m}}{\mathrm{~s}} \sin 70^{\circ} \\
v_{\mathrm{ws}(\mathrm{y})} & =1.2 \frac{\mathrm{~m}}{\mathrm{~s}}(0.9397) \\
v_{\mathrm{ws}(\mathrm{y})} & =1.13 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

| Vector | $x$-component | $y$-component |
| :--- | :--- | :--- |
| $\vec{v}_{\mathrm{cw}}$ | $0.0 \mathrm{~m} / \mathrm{s}$ | $3.2 \mathrm{~m} / \mathrm{s}$ |
| $\vec{v}_{\mathrm{ws}}$ | $0.410 \mathrm{~m} / \mathrm{s}$ | $1.13 \mathrm{~m} / \mathrm{s}$ |
| $\vec{v}_{\mathrm{cs}}$ | $0.410 \mathrm{~m} / \mathrm{s}$ | $4.33 \mathrm{~m} / \mathrm{s}$ |
| $\left\|\vec{v}_{\mathrm{cg}}\right\|^{2}$ | $=v_{\mathrm{cg}(x)}{ }^{2}+v_{\mathrm{cg}(\mathrm{y})}{ }^{2}$ |  |
| $\left\|\vec{v}_{\mathrm{cg}}\right\|^{2}$ | $=\left(0.410 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}+\left(4.33 \frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}$ |  |
| $\left\|\vec{v}_{\mathrm{cg}}\right\|^{2}$ | $=0.1681\left(\frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}+18.75\left(\frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}$ |  |
| $\left\|\vec{v}_{\mathrm{cg}}\right\|^{2}$ | $=18.92\left(\frac{\mathrm{~m}}{\mathrm{~s}}\right)^{2}$ |  |
| $\left\|\vec{v}_{\mathrm{cg}}\right\|$ | $=4.4 \frac{\mathrm{~m}}{\mathrm{~s}}$ |  |
| $\tan \theta_{\mathrm{cg}}$ | $=\frac{4.33 \frac{\mathrm{~m}}{\mathrm{~s}}}{0.410 \frac{\mathrm{~m}}{\mathrm{~s}}}=10.56$ |  |
| $\theta_{\mathrm{cg}}$ | $=\tan ^{-1} 10.56$ |  |
| $\theta_{\mathrm{cg}}$ | $=84.6^{\circ}$ |  |

The velocity of the canoeist relative to the shore is $4.4 \mathrm{~m} / \mathrm{s}\left[\mathrm{N} 5.4^{\circ} \mathrm{E}\right]$.
26. Refer to the scale vector diagrams and the calculations below.

Scale $1 \mathrm{~cm}=2 \mathrm{~km}$


$$
\begin{aligned}
& \Delta t_{1}=50 \mathrm{~min} \times \frac{60 \mathrm{~s}}{1 \mathrm{~min}}=3000 \mathrm{~s} \\
& \Delta \vec{d}_{1}=\vec{v}_{1} \times \Delta t_{1} \\
& \Delta \vec{d}_{1}=2.8 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~W}] \times 3000 \mathrm{~s} \\
& \Delta \vec{d}_{1}=8400 \mathrm{~m}[\mathrm{~W}]=8.4 \mathrm{~km}[\mathrm{~W}] \\
& \Delta t_{2}=30 \mathrm{~min} \times \frac{60 \mathrm{~s}}{1 \mathrm{~min}}=1800 \mathrm{~s} \\
& \Delta \vec{d}_{2}=\vec{v}_{2} \times \Delta t_{2} \\
& \Delta \vec{d}_{2}=3.2 \frac{\mathrm{~m}}{\mathrm{~s}}\left[\mathrm{~N} 30.0^{\circ} \mathrm{W}\right] \times 1800 \mathrm{~s} \\
& \Delta \vec{d}_{2}=5760 \mathrm{~m}\left[\mathrm{~N} 30.0^{\circ} \mathrm{W}\right] \\
& \Delta \vec{d}_{2}=5.76 \mathrm{~km}\left[\mathrm{~N} 30.0^{\circ} \mathrm{W}\right] \\
& \theta_{2}=90^{\circ}-30.0^{\circ}=60^{\circ} \\
& \Delta d_{2(x)}=\left|\Delta \vec{d}_{2}\right| \cos \theta_{2} \\
& \Delta d_{2(x)}=5.76 \mathrm{~km} \cos 60^{\circ} \\
& \Delta d_{2(x)}=5.76 \mathrm{~km}(0.5) \\
& \Delta d_{2(x)}=2.88 \mathrm{~km} \\
& \Delta d_{2(y)}=\left|\Delta \vec{d}_{2}\right| \sin \theta_{2} \\
& \Delta d_{2(y)}=5.76 \mathrm{~km} \sin 60^{\circ} \\
& \Delta d_{2(y)}=5.76 \mathrm{~km}(0.8660) \\
& \Delta d_{2(y)}=4.99 \mathrm{~km}
\end{aligned}
$$

The displacement of the runner is $12 \mathrm{~km}\left[\mathrm{~W} 24^{\circ} \mathrm{N}\right]$.
27. Refer to the scale vectors diagrams and calculations below:

Scale $1 \mathrm{~cm}=4 \mathrm{~km}$

$\Delta \vec{d}_{1}=15 \mathrm{~km}\left[\mathrm{~N} 35^{\circ} \mathrm{E}\right]$
$\Delta \vec{d}_{2}=7.5 \mathrm{~km}\left[\mathrm{~N} 25^{\circ} \mathrm{W}\right]$
$\theta_{1}=90^{\circ}-35^{\circ}=55^{\circ}$
$\Delta d_{1(\mathrm{x})}=\left|\Delta \vec{d}_{1}\right| \cos \theta_{1}$
$\Delta d_{1(x)}=15 \mathrm{~km} \cos 55^{\circ}$
$\Delta d_{1(\mathrm{x})}=15 \mathrm{~km}(0.5736)$
$\Delta d_{1(\mathrm{x})}=8.60 \mathrm{~km}$
$\Delta d_{1(\mathrm{y})}=\left|\Delta \vec{d}_{1}\right| \sin \theta_{1}$
$\Delta d_{1(\mathrm{y})}=15 \mathrm{~km} \sin 55^{\circ}$
$\Delta d_{1(\mathrm{y})}=15 \mathrm{~km}(0.8192)$
$\Delta d_{1(\mathrm{y})}=12.3 \mathrm{~km}$
$\theta_{2}=90^{\circ}-25^{\circ}=65^{\circ}$
$\Delta d_{2(\mathrm{x})}=\left|\Delta \vec{d}_{2}\right| \cos \theta_{2}$
$\Delta d_{2(\mathrm{x})}=7.5 \mathrm{~km} \cos 65^{\circ}$
$\Delta d_{2(\mathrm{x})}=7.5 \mathrm{~km}(0.4226)$
$\Delta d_{2(\mathrm{x})}=3.17 \mathrm{~km}$
$\Delta d_{2(\mathrm{y})}=\left|\Delta \vec{d}_{2}\right| \sin \theta_{2}$
$\Delta d_{2(y)}=7.5 \mathrm{~km} \sin 65^{\circ}$
$\Delta d_{2(\mathrm{y})}=7.5 \mathrm{~km}(0.9063)$
$\Delta d_{2(\mathrm{y})}=6.80 \mathrm{~km}$

| Vector | $x$-component | $y$-component |
| :--- | :--- | :--- |
| $\Delta \vec{d}_{1}$ | 8.60 km | 12.3 km |
| $\Delta \vec{d}_{2}$ | -3.17 km | 6.80 km |
| $\Delta \vec{d}_{\mathrm{T}}$ | 5.43 km | 19.1 km |

$$
\begin{aligned}
\left|\Delta \vec{d}_{\mathrm{T}}\right|^{2} & =\left(\Delta d_{\mathrm{T}(x)}\right)^{2}+\left(\Delta d_{\mathrm{T}(y)}\right)^{2} \\
\left|\Delta \stackrel{\rightharpoonup}{d}_{\mathrm{T}}\right|^{2} & =(5.43 \mathrm{~km})^{2}+(19.1 \mathrm{~km})^{2} \\
\left|\Delta \vec{d}_{\mathrm{T}}\right|^{2} & =29.49 \mathrm{~km}^{2}+364.81 \mathrm{~km}^{2} \\
\left|\Delta \stackrel{\rightharpoonup}{d}_{\mathrm{T}}\right|^{2} & =394.3 \mathrm{~km}^{2} \\
\left|\Delta \vec{d}_{\mathrm{T}}\right| & =19.86 \mathrm{~km}=20 \mathrm{~km} \\
\tan \theta & =\frac{\Delta d_{\mathrm{T}(\mathrm{l})}}{\Delta \mathrm{T}_{\mathrm{I}}(x)} \\
\tan \theta & =\frac{19.1 \mathrm{~km}}{5.43 \mathrm{~km}}=3.52 \\
\theta & =\tan ^{-1} 3.52 \\
\theta & =74^{\circ}
\end{aligned}
$$

(a) The jogger's displacement is $20 \mathrm{~km}\left[\mathrm{~N} 16^{\circ} \mathrm{E}\right]$.
(b) $\vec{v}_{\text {ave }}=\frac{\Delta \vec{d}}{\Delta t}$
$v_{\text {ave }}=\frac{19.86 \mathrm{~km}\left[\mathrm{~N} 16^{\circ} \mathrm{E}\right]}{2.0 \mathrm{~h}}$
$\vec{v}_{\text {ave }}=9.9 \frac{\mathrm{~km}}{\mathrm{~h}}\left[\mathrm{~N} 16^{\circ} \mathrm{E}\right]$
The jogger's average velocity is $9.9 \mathrm{~km}\left[\mathrm{~N} 16^{\circ} \mathrm{E}\right]$.
28.Refer to the scale vector diagrams and calculations below.


$$
\begin{aligned}
& \overrightarrow{v_{\mathrm{i}}}=11.0 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{E}] \\
& -\overrightarrow{v_{\mathrm{i}}}=11.0 \frac{\mathrm{~m}}{\mathrm{~s}}[\mathrm{~W}] \\
& \overrightarrow{v_{\mathrm{f}}}=12.0 \frac{\mathrm{~m}}{\mathrm{~s}}\left[\mathrm{~S} 40.0^{\circ} \mathrm{E}\right]
\end{aligned}
$$

$$
\begin{aligned}
& \theta_{\mathrm{f}}=90^{\circ}-40^{\circ}=50^{\circ} \\
& \nu_{\mathrm{f}(\mathrm{x})}=\left|\vec{v}_{\mathrm{f}}\right| \cos \theta_{\mathrm{f}} \\
& v_{\mathrm{f}(\mathrm{x})}=12.0 \frac{\mathrm{~m}}{\mathrm{~s}} \cos 50^{\circ} \\
& \nu_{\mathrm{f}(\mathrm{x})}=12.0 \frac{\mathrm{~m}}{\mathrm{~s}}(0.6428) \\
& \nu_{\mathrm{f}(\mathrm{x})}=7.71 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& v_{\mathrm{f}(y)}=\left|\stackrel{\rightharpoonup}{v_{\mathrm{f}}}\right| \sin \theta_{\mathrm{f}} \\
& v_{\mathrm{f}(\mathrm{y})}=12.0 \frac{\mathrm{~m}}{\mathrm{~s}} \sin 50^{\circ} \\
& v_{\mathrm{f}}(y)=12.0 \frac{\mathrm{~m}}{\mathrm{~s}}(0.7660) \\
& \nu_{\mathrm{f}(\mathrm{y})}=9.19 \frac{\mathrm{~m}}{\mathrm{~s}}
\end{aligned}
$$

The acceleration of the sailboat is $0.217 \mathrm{~m} / \mathrm{s}^{2}\left[\mathrm{~S} 19.7^{\circ} \mathrm{W}\right]$.
29. (a) Refer to the scale vector diagram below.

$$
\begin{aligned}
& \vec{v}_{\mathrm{ws}}=3.0 \frac{\mathrm{~km}}{\mathrm{~h}} \text { [downstream] } \\
&\left|\overrightarrow{v_{\mathrm{cw}}}\right|=4.0 \frac{\mathrm{~km}}{\mathrm{~h}} \\
& \sin \theta=\overrightarrow{\vec{v}_{\mathrm{vs}}} \\
&\left|\vec{v}_{\mathrm{cw}}\right| \\
& \sin \theta=\frac{3.00 \frac{\mathrm{~km}}{\mathrm{~h}}}{4.0 \frac{\mathrm{~m}}{\mathrm{~h}}} \\
& \sin \theta=0.75 \\
& \theta=\sin ^{-1} 0.75 \\
& \theta=49^{\circ}
\end{aligned}
$$

$\therefore$ He should aim upstream at an angle of $41^{\circ}$ with respect to the river bank.
(b) $\quad\left|\vec{v}_{\mathrm{cw}}\right|^{2}=\left|\vec{v}_{\mathrm{cs}}\right|^{2}+\left|\vec{v}_{\mathrm{ws}}\right|^{2}$
$\left(4.0 \frac{\mathrm{~km}}{\mathrm{~h}}\right)^{2}=\left|\vec{v}_{\mathrm{cs}}\right|^{2}+\left(3.0 \frac{\mathrm{~km}}{\mathrm{~h}}\right)^{2}$

$$
\left|\vec{v}_{\mathrm{cs}}\right|^{2}=16\left(\frac{\mathrm{~km}}{\mathrm{~h}}\right)^{2}-9\left(\frac{\mathrm{~km}}{\mathrm{~h}}\right)^{2}
$$

$$
\left|\overrightarrow{v_{\mathrm{cs}}}\right|^{2}=7\left(\frac{\mathrm{~km}}{\mathrm{~h}}\right)^{2}
$$

$$
v_{\mathrm{cs}}=2.65 \frac{\mathrm{~km}}{\mathrm{~h}}
$$

$\therefore$ His velocity relative to the shore is $2.65 \mathrm{~km} / \mathrm{h}$.

$$
\begin{aligned}
\Delta t & =\frac{\Delta \vec{d}}{\overrightarrow{v_{\mathrm{cs}}}} \\
\Delta t & =\frac{0.10 \mathrm{~km}}{2.65 \frac{\mathrm{~km}}{\mathrm{~h}}} \\
\Delta t & =0.0377 \mathrm{~h} \times \frac{60 \mathrm{~min}}{1 \mathrm{~h}} \\
\therefore \Delta t & =2.3 \mathrm{~min}
\end{aligned}
$$

The trip will take him 2.3 min .


