Sound Waves and Electromagnetic Radiation

Practice Problem Solutions

Student Textbook page 390

1. **Frame the Problem**
   - The speed of sound can be calculated from the temperature.
   - At a temperature of 0°C and a pressure of 101 kPa, the speed of sound in air is 331 m/s.
   - For each 1°C rise in temperature, the speed of sound increases by 0.59 m/s.

   Identify the Goal
   The speed of sound, v

   Variables and Constants

Known	Unknown
(a) $T_C = -15°C$	v
(b) $T_C = 15°C$	
(c) $T_C = 25°C$	
(d) $T_C = 33°C$	

   Strategy
   Use the formula for the velocity (speed) of sound in air.
   Substitute the known temperature and calculate the speed.

   (a) $\quad v = 331 + 0.59\ T_C$
   $$v = 331\ \text{m/s} + 0.59\tfrac{\text{m/s}}{°C}(-15°C)$$
   $$v = 322\ \text{m/s}$$
   The speed of sound is 322 m/s or 3.2×10^2 m/s.

   Validate
   The temperature is a little below zero, so the speed of sound should be a little less than 331 m/s.
   Similarly,
   (b) speed of sound $= 331.3\ \text{m/s} + 0.59\tfrac{\text{m/s}}{°C} \times 15°C = 340\ \text{m/s} = 3.4 \times 10^2$ m/s

   (c) speed of sound $= 331.3\ \text{m/s} + 0.59\tfrac{\text{m/s}}{°C} \times 25°C = 346\ \text{m/s} = 3.5 \times 10^2$ m/s

   (d) speed of sound $= 331.3\ \text{m/s} + 0.59\tfrac{\text{m/s}}{°C} \times 33°C = 350\ \text{m/s} = 3.5 \times 10^2$ m/s

2. **Frame the Problem**
   - The speed of sound depends on air temperature.
   - If you know the speed, you should be able to calculate the temperature.

   Identify the Goal
   The goal is the temperature in °C.

Variables and Constants

Known	Unknown
(a) $v = 352$ m/s	T_C
(b) $v = 338$ m/s	
(c) $v = 334$ m/s	
(d) $v = 319$ m/s	

Strategy

Use the temperature equation and solve for T_C.

(a) 352 m/s $= 331$ m/s $+ 0.59\,T_C$

$\quad\quad 0.59\,T_C = 352$ m/s $- 331$ m/s

$\quad\quad\quad\quad\quad = 21$ m/s

$\quad\quad\quad T_C = \dfrac{21 \text{ m/s}}{0.59 \frac{\text{m/s}}{°C}}$

$\quad\quad\quad\quad\quad = 36°C$

The air temperature is $36°C$.

Validate

Room air temperature has a speed of sound of about 344 m/s. A speed of 352 m/s requires a temperature a little higher than room temperature.

Similarly:

(b) $T_C = \dfrac{(338 \text{ m/s} - 331 \text{ m/s})}{0.59 \frac{\text{m/s}}{°C}}$

$\quad\quad\quad = 12°C$

(c) $T_C = \dfrac{(334 \text{ m/s} - 331 \text{ m/s})}{0.59 \frac{\text{m/s}}{°C}}$

$\quad\quad\quad = 5°C$

(d) $T_C = \dfrac{(318 \text{ m/s} - 331 \text{ m/s})}{0.59 \frac{\text{m/s}}{°C}}$

$\quad\quad\quad = -22°C$

3. (a) Frame the Problem

- Sound travels to the iceberg and back.
- The time for the echo depends on the distance to the iceberg and the speed of sound.
- The sound's speed depends on the temperature, which is given.

Identify the Goal

The distance to the iceberg

Variables and Constants

Known	Unknown
$\Delta t = 3.8$ s	Δd
$T_C = -12°C$	

Strategy

Use the temperature and speed of sound formula to find the speed of sound.

Use $v = \dfrac{\Delta d}{\Delta t}$ to find the distance.

The sound travelled to the iceberg and back, so distance to iceberg is $\dfrac{\Delta d}{2}$.

$$v = 331 + 0.59 \; T_C$$
$$= 331 \text{ m/s} + 0.59 \tfrac{\text{m/s}}{°C}(-12°C)$$
$$= 333.5 \text{ m/s}$$
$$v = \frac{\Delta d}{\Delta t}$$
$$\Delta d = v\Delta t$$
$$= 333.5 \text{ m/s} \times 3.8 \text{ s}$$
$$= 1267 \text{ m}$$
$$\text{distance to iceberg} = \frac{\Delta d}{2}$$
$$= 633 \text{ m}$$

The distance to iceberg is 6.3×10^2 m.

Validate

Sound travels about 1 km every 3 seconds. The one-way trip took a little less than 2 seconds, so the iceberg is a little less than 1 km away.

(b) We have accounted for the temperature. Perhaps the atmospheric pressure was not 101 kPa. If the weather was foggy, the speed of sound in very humid air may differ from the standard 331 m/s + 0.59 T_C.

4. Frame the Problem

- Sound travels to the fish and back.
- The time for the echo depends on the distance to the fish and the speed of sound in water, 1482 m/s in a freshwater lake.

Identify the Goal

The distance to the fish

Variables and Constants

Known	Unknown
$D = 35$ m	Δt
$v = 1482$ m/s	

Strategy

Double the distance to the fish to find the round trip time of the sound pulse:
$\Delta d = 2D$.

Use $v = \frac{\Delta d}{\Delta t}$ to find the time.

$$\Delta d = 2 \times D$$
$$= 70 \text{ m}$$
$$v = \frac{\Delta d}{\Delta t}$$
$$\Delta t = \frac{\Delta d}{v}$$
$$= \frac{70 \text{ m}}{1482 \text{ m/s}}$$
$$= 0.047 \text{ s}$$

The time delay was 0.047 s.

Validate

Sound travels very quickly in water, over one kilometre per second. The time delay should be very short.

5. Frame the Problem
- Sound travels to the far wall of the stadium and back.
- The time for the echo depends on the distance to the end of the stadium and the speed of sound.
- The sound's speed depends on the temperature, which is given.

Identify the Goal
Distance to the far end of the stadium.

Variables and Constants

Known	Unknown
$\Delta t = 1.2$ s	Δd
$T_C = 12°C$	l

Strategy
Use the temperature and speed of sound formula to find the speed of sound.
Use $v = \frac{\Delta d}{\Delta t}$ to find the round trip distance.
Divide Δd by 2 to calculate length of stadium.

$$v = 331 \text{ m/s} + 0.59 \ T_C$$
$$= 331 \text{ m/s} + 0.59 \ \tfrac{\text{m/s}}{°C}(12°C)$$
$$= 338.1 \text{ m/s}$$
$$v = \frac{\Delta d}{\Delta t}$$
$$\Delta d = v\Delta t$$
$$= 338.1 \text{ m/s} \times 1.2 \text{ s}$$
$$= 405.7 \text{ m}$$
$$l = \frac{\Delta d}{2}$$
$$= 202.8 \text{ m}$$

The length of the stadium is 2.0×10^2 m.

Validate
Sound travels about 1 km every 3 s. The one-way trip took a little over half a second, so the length of the stadium should be a little over one sixth of a kilometre.

6. (a) Frame the Problem
- Time of travel depends on speed of sound and distance. Distance is given.
- The speed of sound depends on the temperature, which is given.
$$v = \frac{\Delta d}{\Delta t}$$
The time of flight, Δt, of the sound

Variables and Constants

Known	Unknown
$T_C = 22°C$	v
$\Delta d = 2.0$ km $= 2000$ m	Δt

Strategy
Use the temperature and speed of sound formula to find the speed of sound.
Use $v = \frac{\Delta d}{\Delta t}$ to find the time.
Use distance in metres.

$$v = 331 \text{ m/s} + 0.59 \; T_C$$
$$= 331 \text{ m/s} + 0.59 \tfrac{\text{m/s}}{°C} (22°C)$$
$$= 344.0 \text{ m/s}$$
$$v = \frac{\Delta d}{\Delta t}$$
$$\Delta t = \frac{\Delta d}{v}$$
$$= \frac{2000 \text{ m}}{344 \text{ m/s}}$$
$$= 5.8 \text{ s}$$

Sound would take 5.8 s to travel 2.0 km at 22°C.

Validate

Sound travels about 1 km every 3 s. It would take about 6 s to travel 2 km.

(b) Similarly, if speed is 3.0×10^8 m/s, time to travel 2 km would be

$$v = \frac{\Delta d}{\Delta t}$$
$$\Delta t = \frac{\Delta d}{v}$$
$$= \frac{2000 \text{ m}}{3.0 \times 10^8 \text{ m/s}}$$
$$= 6.7 \times 10^{-7} \text{ s}$$

(c) Frame the Problem

- After the lightning hits the church, the light travels 2 km to your eyes.
- The sound of the thunder travels 2 km to your ears.
- If 8 s elapses in between seeing the lightning hit the church and hearing the thunder, then the actual time of travel of the sound was 8 s plus the time it took for light to reach your eyes. Part (b) tells us that this time is so short we can neglect it.

Identify the Goal

The distance, Δd, to the church

Variables and Constants

Known	Unknown
$T_C = 22°C$	Δd
$v = 344$ m/s (from part (a)	
$\Delta t = 8.0$ s	

Strategy

Since temperature is the same, use speed of sound from part (a).

Use $v = \frac{\Delta d}{\Delta t}$ to find the distance.

$$\Delta d = v\Delta t$$
$$= 340 \text{ m/s} \times 8.0 \text{ s}$$
$$= 2620 \text{ m}$$

The church was 2.6 km away.

Validate

Sound takes about 3 s to cover 1 km. So in 8 s, the sound would travel a little less than 3 km.

Practice Problem Solutions

7. Frame the Problem
- Make a sketch of the problem. Refer to the student textbook Model Problem "Index of Refraction"
- The angles of incidence and refraction are known
- The index of refraction relates the angles of incidence and refraction when the incident medium is air
- Air is the incident medium

Identify the Goal
The index of refraction of an unknown material
Identify the unknown material

Variables and Constants

Known	Unknown
incident medium: air	n
$\theta_I = 59°$	material
$\theta_R = 49°$	

Strategy

Use Snell's law.

Since the incident medium is air, the constant is the index of refraction, n, of the liquid.

Substitute the known values.

Calculations

$$\frac{\sin \theta_I}{\sin \theta_R} = \text{a constant}$$

$$n = \frac{\sin \theta_I}{\sin \theta_R}$$

$$n = \frac{\sin 59°}{\sin 41°}$$

$$n = 1.3065$$

The unknown index of refraction is 1.31. Accordingly, the material is ice.
(see Table 9.2)

Validate
The absence of units is in agreement with the unitless nature of the index of refraction. The value is between one and two, which is very reasonable for an index of refraction.

8. Frame the Problem
- Make a sketch of the problem.
- The angles of incidence are known.
- The refractive medium is known.
- The index of refraction relates the angles of incidence and refraction when the incident medium is air.
- Air is the incident medium.

Identify the Goal
The angle of refraction in zircon crystal

Variables and Constants

Known	Unknown
Incident medium: air	θ_R
Refractive medium: zircon	
$\theta_I = 72.0°$	
$n = 1.92$ (from Table 9.2)	

Strategy	**Calculations**
Use Snell's constant.	$\dfrac{\sin \theta_i}{\sin \theta_R} = $ a constant
Since the incident medium is air, the constant is the index of refraction, n, of zircon.	$\sin \theta_R = \dfrac{\sin \theta_i}{n}$
Substitute the known values.	$\sin \theta_R = \dfrac{\sin 72°}{1.92}$
Rearrange the equation to solve for θ_R.	$\theta_R = \sin^{-1}(0.4953)$
The angle of refraction is 29.7°.	$\theta_R = 29.7°$

Validate
The angle is stated in degrees. The angle of refraction is less than the angle of incidence. This makes sense because when light enters a denser medium, it slows down and bends toward the normal.

9. Frame the Problem
- Make a sketch of the problem. Refer to the student textbook model problem on page 511.
- The angles of refraction is known.
- The refractive medium is known.
- The index of refraction relates the angles of incidence and refraction when the incident medium is air.
- Air is the incident medium.

Identify the Goal
The angle of incidence in ethyl alcohol

Variables and Constants

Known	**Unknown**
Incident medium: air	θ_i
Refractive medium: ethyl alcohol	
$\theta_R = 35°$	
$n = 1.362$ (from Table 9.2)	

Strategy	**Calculations**
Use Snell's constant.	$\dfrac{\sin \theta_i}{\sin \theta_R} = $ a constant
Since the incident medium is air, the constant is the index of refraction, n, of zircon.	$\dfrac{\sin \theta_i}{\sin \theta_R} = n$
Rearrange the equation to solve for θ_R.	$\sin \theta_i = n \sin \theta_R$
Substitute the known values.	$\sin \theta_i = 1.362 \times \sin 35°$
The angle of incidence is index 51°.	$\theta_i = \sin^{-1}(0.0781)$
	$\theta_i = 51.4°$

Validate
The angle is stated in degrees. The angle of incidence is greater than the angle of refraction. This is realistic because when light enters a denser medium, it slows down and bends toward the normal.

Practice Problem Solutions

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10. **Frame the Problem**
    - Make a sketch of the problem.
    - Light travels from air, an optically less dense medium, into ethyl alcohol, an optically more dense medium.
    - The refracted ray should bend toward the normal line.
    - You can use Snell's law to determine the extent of the bending of the refracted ray.

    Identify the Goal
    The angle of refraction, θ_R, in ethyl alcohol

    Variables and Constants

Known	Implied	Unknown
$\theta_i = 60°$	$n_i = 1.00$	θ_R
	$n_R = 1.362$	

Strategy	Calculations
Use Snell's law to solve problem.	$n_i \sin \theta_i = n_R \sin \theta_R$
Rearrange the equation to solve for θ_R.	$\sin \theta_R = \frac{n_i \sin \theta_i}{n_R}$
	$\theta_R = \sin^{-1}\left(\frac{n_i \sin \theta_i}{n_R}\right)$
Substitute the known values.	$\theta_R = \sin^{-1}\left(\frac{1.00(\sin 60.0°)}{1.362}\right)$
	$\theta_R = 39.4899$

    The angle of refraction is 39.5°.

    Validate
    The angle is stated in degrees. The angle of refraction is less than the angle of incidence. This is realistic because when light enters a denser medium it slows down and bends towards the normal.

11. **Frame the Problem**
    - Make a sketch of the problem. Refer to the model problem "Finding the Angle of Refraction" in the student textbook.
    - Light travels from ethyl alcohol, an optically more dense medium, into air, an optically less dense medium.
    - The refracted ray should bend away from the normal line (meaning also that the angle of the incident ray should be less than the angle of the refracted ray).
    - You can use Snell's law to determine the extent of the bending of the refracted ray.

    Identify the Goal
    The angle of incidence, θ_i, from ethyl alcohol into air.

    Identify the Variables

Known	Implied	Unknown
$\theta_R = 44.5°$	$n_i = 1.362$	θ_i
	$n_R = 1.00$	

Develop a Strategy

Use Snell's law to solve the problem.

Calculations

$$n_i \sin \theta_i = n_R \sin \theta_R$$

$$\sin \theta_i = \frac{n_R \sin \theta_R}{n_i}$$

$$\sin \theta_i = \frac{(1.00)\sin 44.5°}{1.362}$$

$$\sin \theta_i = 0.5146$$

$$\theta_i = \sin^{-1}(0.5146)$$

$$\theta_i = 30.97°$$

$$\theta_i \cong 31.0°$$

The angle of incidence is 31.0°.

Validate the Solution

The angle is in degrees as required. It is less than the angle of refraction, which is expected for situations when a light beam travels from a medium of higher index to one of lower index (as the light enters the lower density medium, it travels further away from the normal). So, the answer is reasonable.

Practice Problem Solutions

Student Textbook page 410

12. **Frame the Problem**
    - Make a sketch of the problem. Label all of the media, angles and rays. See model problem on page 536 of the student textbook.
    - Light is travelling from an optically more dense medium to an optically less dense medium.
    - The critical angle of the incidence corresponds to an angle of refraction of 90°.
    - The needed indices of refraction are listed in Table 9.2.

    Identify the Goal
    Calculate the critical angle, θ_c for ethyl alcohol

    Variables and Constants

Implied	Unknown
$n_{air} = 1.00$	$\theta_{c(ethyl\ alcohol)}$
$n_{ethyl\ alcohol} = 1.362$	

    Strategy

    Use Snell's law.

    The critical angle of incidence, $\theta_{c(ethyl\ alcohol)}$, occurs when the angle of refraction is exactly 90°.

    $\theta_i = \theta_{c(ethyl\ alcohol)}$ and $\theta_R = 90°$.

    Rearrange for $\theta_{c(ethyl\ alcohol)}$.

    Calculations

    $$n_i \sin \theta_i = n_R \sin \theta_R$$

    $$n_i \sin \theta_{c(ethyl\ alcohol)} = n_R \sin 90°$$

    $$\theta_{c(ethyl\ alcohol)} = \sin^{-1} \frac{n_R \sin 90°}{n_i}$$

    $$\theta_{c(ethyl\ alcohol)} = \sin^{-1}\left(\frac{1.00(1.00)}{1.362}\right)$$

    $$\theta_{c(ethyl\ alcohol)} = 47.2408°$$

    The critical angle for ethyl alcohol is 47.2°.

Validate

The angle of incidence is between 0° and 90°.

13. Frame the Problem
- Make a sketch of the problem like the diagram on page 536 of the student textbook. Label all of the media, angles and rays.
- Light is travelling from an optically more dense medium to an optically less dense medium (in this case plastic to water).
- The critical angle of the incidence corresponds to an angle of refraction of 90°.
- The needed indices of refraction are listed in Table 9.2.

Identify the Goal

Calculate the critical angle, θ_c for plastic if is immersed in water.

(a) calculate $n_{plastic}$

(b) calculate $\theta_{c(plastic)}$

Variables and Constants

Known	Implied	Unknown
$n_{air} = 1.00$	$\theta_{c(air)} = 40°$	$n_{plastic}$
$n_{water} = 1.333$		$\theta_{c(plastic)}$
$\theta_{water} = 90°$		

Strategy

Use Snell's law.

The critical angle of incidence, $\theta_{c(plastic)}$, occurs when the angle of refraction is exactly 90°.

$\theta_i = \theta_{c(plastic)}$ and $\theta_R = 90°$.

$n_R = n_{air}$

Rearrange for $n_i = n_{plastic}$

Calculations

$n_i \sin \theta_i = n_R \sin \theta_R$

$n_i \sin \theta_{c(plastic)} = n_R \sin 90°$

$n_i = \dfrac{n_R \sin 90°}{\sin \theta_{c(plastic)i}}$

$n_i = \dfrac{1.00(1.00)}{\sin 40°}$

$n_i = 1.555$

The optical density (index of refraction) for the plastic is 1.556.

Use Snell's law to solve for the angle of incidence from the plastic.

$n_i = n_{plastic}$, $\theta_i = \theta_{c(plastic)}$

$n_R = n_{water}$, $\theta_R = \theta_{water} = 90°$

Rearrange for $\theta_i = \theta_{c(plastic)}$

$n_i \sin \theta_i = n_R \sin \theta_R$

$\theta_{c(plastic)} = \sin^{-1}\left(\dfrac{n_R \sin \theta_R}{n_i}\right)$

$\theta_{c(plastic)} = \sin^{-1}\left(\dfrac{1.333 \sin 90°}{1.556}\right)$

$\theta_{c(plastic)} = 58.9°$

The critical angle for the plastic, when it is immersed in water, is 58.9°.

Validate

The critical angle for the plastic, when it is immersed in water, is greater than the critical angle for plastic immersed in air. This is realistic because the change in speed of the mediums is less, meaning the light will not bend as drastically when moving from one medium to the other.

14. Frame the Problem

- Make a sketch of the problem like the diagram on page 409 of the student text-book. Label all of the media, angles and rays.
- Light is travelling from an optically more dense medium to an optically less dense medium (in this case core to cladding).
- The critical angle of the incidence corresponds to an angle of refraction of 90°.
- The needed indices of refraction are listed in Table 9.2.

Identify the Goal
Calculate the critical angle between the core-cladding interface

Variables and Constants

Implied	Unknown
$n_{core} = 1.50$	$\theta_{c(core)}$
$n_{cladding} = 1.47$	
$\theta_{R(cladding)} = 90°$	

Strategy	Calculations
Use Snell's law.	$n_i \sin \theta_i = n_R \sin \theta_R$
The critical angle of incidence, $\theta_{c(core)}$, occurs when the angle of refraction is exactly 90°.	$n_i \sin \theta_{c(core)} = n_R \sin 90°$
$\theta_i = \theta_{c(core)}$ and $\theta_R = 90°$.	
$n_R = n_{cladding}$	
Rearrange for , $\theta_i = \theta_{c(core)}$	$\theta_{c(core)} = \sin^{-1}(\frac{n_R \sin 90°}{n_i})$
Substitute in for the values.	$\theta_{c(core)} = \sin^{-1}(\frac{1.47(1.00)}{1.50})$
	$\theta_{c(core)} = 78.5°$

The critical angle for the plastic, when it is immersed in water, is 78.5°.

Validate
The critical angle for the core-cladding interface is quite high. This is realistic because optical fibres need to trap the maximum amount of light.

15. Frame the Problem

- Make a sketch of the problem. Specifically draw a triangle to show the distance away from the pools edge (across the top), the depth of the eye (along the side) and the angle from the eye to the surface of the water. This will represent the critical angle.
- Light is travelling from an optically more dense medium to an optically less dense medium (in this case, water to air).
- The critical angle of the incidence corresponds to an angle of refraction of 90°.
- The needed indices of refraction are listed in Table 9.2.

Identify the Goal
(a) Calculate the critical angle, θ_c, for light passing from water to air.

(b) Calculate the depth of the eye.

Variables and Constants

Implied	Known	Unknown
$n_{air} = 1.00$	distance from edge = 3.0 m	$\theta_{c(water)}$
$n_{water} = 1.333$		d
$\theta_{water} = 90°$		

Strategy	Calculations
Use Snell's law to solve for the critical angle.	$n_i \sin \theta_i = n_R \sin \theta_R$
The critical angle of incidence, $\theta_{C(water)}$, occurs when the angle of refraction is exactly 90°.	$n_i \sin \theta_{C(cwater)} = n_R \sin 90°$

$\theta_i = \theta_{C(water)}$ and $\theta_R = 90°$

$n_R = n_{air}$

Rearrange for θ_i and substitute in the values.

$$\sin \theta_{C_{(water)}} = \frac{n_R \sin 90°}{n_i}$$

$$\theta_{C_{(water)}} = \sin^{-1}(\frac{n_R \sin \theta_R}{n_i})$$

$$\theta_{C_{(water)}} = \sin^{-1}(\frac{100 \sin}{1.333} 90°)$$

$$\theta_{C_{(water)}} = 48.6°$$

$$\theta_{(triangle)} = 90.0° - 48.6° = 41.4°$$

The critical angle is 48.6°.

Use the critical angle to find the angle in the trig. triangle. This is the angle between the line your eye makes with the edge of the pool and the surface of the water.	$\tan \theta = \frac{\text{depth}}{\text{distance from edge}}$

Use trigonometry to solve for the depth.

Rearrange equation to solve for depth.	depth = $\tan \theta$ (distance from edge)
Substitute in for values.	depth = $\tan 48.6°$ (distance from edge)
	depth = 2.6 m

The eye must be 2.6 m below the surface for total internal reflection to occur.

Practice Problem Solutions

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16. Frame the Problem
- The pipe and cylinder form closed air columns.
- The distance between antinodes is $\frac{\lambda}{2}$.

Identify the Goal
(a) the wavelength, λ, of the sound
(b) the next resonance length, L_3

Variables and Constants

Known	Unknown
$L_1 = 17$ cm	λ
$L_2 = 51$ cm	L_3

Strategy
The difference between L_2 and L_1 is half of the wavelength of the sound.
Add another half wavelength to calculate the next resonance length.

(a) $\Delta l = l_2 - l_1$
      $= 51 \text{ cm} - 17 \text{ cm}$
      $= 34 \text{ cm}$
   $\lambda = 2L$
      $= 2 \times 34 \text{ cm}$
      $= 68 \text{ cm}$

(b) $l_3 = l_2 + \Delta l$
      $= 51 \text{ cm} + 34 \text{ cm}$
      $= 85 \text{ cm}$

The wavelength is 68 cm, and the next resonance length is 85 cm.

Validate
The third resonance should be at $\frac{5\lambda}{4}$. So the third resonance should be
$\frac{(5 \times 68 \text{ cm})}{4} = 85 \text{ cm}$, as found in part (b).

17. Frame the Problem
- In closed air columns, the first resonance length is $\frac{\lambda}{4}$. The subsequent resonances are $\frac{\lambda}{2}$ apart.
- In open air columns, first resonance length is $\frac{\lambda}{2}$. The next resonances are $\frac{\lambda}{2}$ apart.
- The first resonance is given, so λ can be calculated in each case.

Identify the Goal
The first three resonances for (a) a closed air column and (b) an open air column

Variables and Constants
Known   **Unknown**
$L_1 = 32 \text{ cm}$ L_2

                              L_3

Strategy
Calculate the wavelength from the first resonance length, then keep adding half the wavelength to find the second and third resonance lengths.

(a) $L_1 = 32 \text{ cm}$
   for closed air column, $\lambda = 4L_1$
                        $= 4 \times 32 \text{ cm}$
                        $= 128 \text{ cm}$
                 $\frac{1}{2}\lambda = 64 \text{ cm}$
                  $L_2 = 32 \text{ cm} + 64 \text{ cm}$
                      $= 96 \text{ cm}$
                  $L_3 = 96 \text{ cm} + 64 \text{ cm}$
                      $= 160 \text{ cm}$

The second and third resonance lengths are 96 cm and 160 cm.

Validate
The third resonance length should be at $\frac{5\lambda}{4} = 1.25 \times 128 \text{ cm} = 160 \text{ cm}$

(b) $$L_1 = 32 \text{ cm}$$

for open air column, $\lambda = 2L_1$
$$= 2 \times 32 \text{ cm}$$
$$= 64 \text{ cm}$$
$$\tfrac{1}{2}\lambda = 32 \text{ cm}$$
$$L_2 = 32 \text{ cm} + 32 \text{ cm}$$
$$= 64 \text{ cm}$$
$$L_3 = 64 \text{ cm} + 32 \text{ cm}$$
$$= 96 \text{ cm}$$

The second and third resonance lengths are 64 cm and 96 cm.

Validate

The third resonance length should be at $\frac{3\lambda}{2} = 1.5 \times 64 \text{ cm} = 96 \text{ cm}$

18. Frame the Problem

- The problem relates to a closed air column.
- Resonances are at $\frac{\lambda}{4}$, $\frac{3\lambda}{4}$, and $\frac{5\lambda}{4}$
- The third resonance is given, so we can calculate the others.

Variables and Constants

Known	Unknown
$L_3 = 95 \text{ cm}$	λ
	L_1
	L_2

Strategy

From the third resonance length, calculate λ.
Calculate the other resonance lengths using λ.

(a) $$l_3 = 95 \text{ cm}$$

for closed air column, $\frac{5}{4}\lambda = L_3$
$$= 95 \text{ cm}$$
Therefore $\lambda = \frac{4}{5} \times 95 \text{ cm}$
$$= 76 \text{ cm}$$
$$L_1 = \tfrac{1}{4} \times 76 \text{ cm}$$
$$= 19 \text{ cm}$$
$$L_2 = \frac{3\lambda}{4}$$
$$= \tfrac{3}{4} \times 76 \text{ cm}$$
$$= 57 \text{ cm}$$

Validate

For a closed air column, third resonance length should be
$L_1 + \lambda = 19 \text{ cm} + 76 \text{ cm} = 95 \text{ cm}$ as given.

19. Frame the Problem

- The problem involves an open air column.
- Resonances are at $\frac{\lambda}{2}$, λ, and $\frac{3\lambda}{2}$.
- The second resonance is given, so we can calculate the others.

Variables and Constants

Known	Unknown
$L_2 = 95$ cm	λ
	L_1
	L_3

Strategy

From the second resonance length, calculate λ.
Calculate the other resonance lengths using λ.

(a)
$$l_2 = 64 \text{ cm}$$
$$\text{for open air column, } \lambda = L_2$$
$$\text{therefore } \lambda = 64 \text{ cm}$$
$$L_1 = \tfrac{1}{2} \times 64 \text{ cm}$$
$$= 32 \text{ cm}$$
$$L_3 = \tfrac{3}{2}\lambda$$
$$= 96 \text{ cm}$$

Validate

For an open air column, the third resonance should be triple the first:
3×32 cm $= 96$ cm.

20. (a) Frame the Problem

- The problem involves an open air column.
- The fundamental is $L_1 = \frac{\lambda}{2}$, so length of pipe depends on λ.
- The temperature T_C determines speed of sound, v.
- v and f determine λ, which determines length L_1.

Identify the Goal

The length of the pipe, L_1, the first resonant length of the open air column

Variables and Constants

Known	Unknown
$f = 128$ Hz	L_1
$T_C = 22°C$	v

Strategy

Use temperature to find velocity.
Use the wave equation to find λ.
Use λ to find the first resonant length L_1, the length of the pipe.
$$v = 331 \text{ m/s} + 0.59(T_C)$$
$$= 344 \text{ m/s}$$
$$\lambda = \frac{v}{f}$$
$$= \frac{344 \text{ m/s}}{128 \text{ Hz}}$$
$$= 2.69 \text{ m}$$
$$l_1 = \tfrac{1}{2}\lambda$$
$$= 1.34 \text{ m}$$
The length of the pipe will be 1.34 m.

Validate

The speed of sound is about 340 m/s. A wavelength of a 128 Hz note is about $\frac{(340 \text{ m/s})}{(128 \text{ Hz})}$ or between 2.5 m and 3 m. The length of the open air column should be between 1.25 m and 1.5 m.

(b) Frame the Problem
- This problem involves a closed air column.
- The length of pipe and speed of sound are given (from part (a)).
- The length of pipe determines wavelength.
- The wavelength and speed of sound determine frequency.

Identify the Goal
Frequency, f, of note

Variables and Constants

Known	Unknown
$L_1 = 1.34$ m	λ
$v = 344$ m/s	f

Strategy
The length of the column is $L_1 = \frac{\lambda}{4}$, so multiply length by 4 to calculate λ.
Use the wave equation to calculate frequency.

$L_1 = 1.34$ m

$\lambda = 4 \times L_1$

$\quad = 5.36$ m

$v = f\lambda$

$f = \frac{v}{\lambda}$

$\quad = \frac{344 \text{ m/s}}{5.36 \text{ m}}$

$\quad = 64$ Hz

The organ pipe will produce a 64 Hz tone.

Validate
The same air column as in part (a) now holds a note whose wavelength is four times the length of the column, not twice the length. The frequency should therefore be half the frequency of the note in part (a).

Practice Problem Solutions

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21. Frame the Problem
- This problems involves an open air column with antinodes at each end.
- The harmonics of open air column are integer multiples of the resonant frequency.

Identify the Goal
The frequencies of the second and third harmonics, f_2 and f_3

Variables and Constants

Known	Unknown
$f_1 = 256$ Hz	f_2
	f_3

Strategy

Multiply fundamental frequency by 2 and 3
$f_2 = f_1 \times 2 = 512$ Hz
$f_3 = f_1 \times 3 = 768$ Hz
The second harmonic (first overtone) is 512 Hz. The third harmonic (second overtone) is 768 Hz.

Validate
The first overtone is double the frequency and thus an octave higher, as expected.

22. **Frame the Problem**
    - This problem involves an open air column whose fundamental length is $L_1 = \frac{\lambda}{2}$.
    - The first overtone is twice the fundamental frequency; second overtone is three times the fundamental frequency
    - Speed of sound in air is needed to determine the frequency.

    Identify the Goal
    The first three harmonics for a bugle, f_1, f_2, and f_3

    Variables and Constants

Known	Unknown
$L_1 = 2.65$ m	λ
assume $v = 344$ m/s	f_1, f_2, and f_3

    Strategy
    Calculate the fundamental wavelength from the first resonance length.
    Use the wave equation to find fundamental frequency, f_1
    Multiply f_1 by 2 and 3 to find overtones.

    $\lambda = 2L_1$
    $\quad = 2 \times 2.65$ m
    $\quad = 5.30$ m

    $v = f_1\lambda$
    $f_1 = \frac{v}{\lambda}$
    $\quad = \frac{344 \text{ m/s}}{5.30 \text{ m}}$
    $\quad = 64.9$ Hz

    $f_2 = 2 \times f_1$
    $\quad = 129.8$ Hz

    $f_3 = 3 \times f_1$
    $\quad = 194.7$ Hz

    (a) The lowest note a bugle can play (when speed of sound is 344 m/s) is 64.9 Hz.

    (b) The next two higher frequencies that will produce resonances are 130 Hz and 195 Hz

    Validate
    (a) The speed of sound and the frequency should determine the wavelength. Dividing the wavelength by 2 should determine the bugle's length. Check: $\frac{(344 \text{ m/s})/(64.9 \text{ Hz})}{2} = 2.65$ as given in question.
    (b) The second overtone should be the fundamental + second overtone for an open pipe. Check: 64.9 Hz + 129.8 Hz = 194.7 Hz, as calculated.

23. **Frame the Problem**
    - This problem involves an open air column.
    - The first overtone (second harmonic) is double the resonant frequency.
    - The length of the tube for the fundamental (first harmonic) is half the wavelength.
    - The wavelength can be found from the frequency and the speed of sound.

Variables and Constants

Known	Unknown
$f_1 = 87.3$ Hz	f_2
$v = 344$ m/s	λ_c

Strategy

Double first harmonic to get second harmonic.
Use universal wave equation to find fundamental wavelength.
First resonant length is $L_1 = \frac{\lambda}{2}$

(a) $f_1 = 87.3$ Hz
$f_2 = f_1 \times 2$
$= 176.6$ Hz

(b) $v = f\lambda$
$\lambda = \frac{v}{f_1}$
$= \frac{344 \text{ m/s}}{87.3 \text{ Hz}}$
$= 3.94$ m
$l = \frac{\lambda}{2}$
$= 1.97$ m

(a) The second harmonic is 177 Hz.

(b) The length of the tube for playing the fundamental is 1.98 m. To play this note, lips must be as relaxed as possible, and the slide should be pushed out to its maximum length.

Validate

Doubling the length of the tube determines the wavelength; multiplying this figure by the frequency calculates the speed of sound. Check: 1.97 m $\times$ 2 $\times$ 87.3 Hz = 344 m/s, as given.

Practice Problem Solutions

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24. (a) Frequencies are close enough together so that person would hear a note of approximately 516 Hz fluctuating in intensity.

(b) **Frame the Problem**
- Two forks of slightly different frequency are sounding at the same time.
- Beats will be heard.

Identify the Goal
Beat frequency, f_{beat}

Variables and Constants

Known	Unknown
$f_1 = 512$ Hz	f_{beat}
$f_2 = 518$ Hz	

Strategy
Beat frequency is the difference in frequencies.
$f_{beat} = |518 \text{ Hz} - 512 \text{ Hz}|$
$= 6$ Hz
The beat frequency is 6 Hz.

25. Frame the Problem
- Two forks of slightly different frequency are sounding at the same time.
- Beats will be heard.
- The number of beats per second equals the difference in the frequencies.

Identify the Goal
Number of beats, N, heard in 3.0 s

Variables and Constants

Known	Unknown
$f_1 = 440$ Hz	f_{beat}
$f_2 = 437$ Hz	N

Strategy
Number of beats per second, f_{beat}, is the difference in frequencies.
Multiply f_{beat} by the time interval to calculate the total number of beats.

$$f_{beat} = |f_1 - f_2|$$
$$= |440 \text{ Hz} - 337 \text{ Hz}|$$
$$= 3 \text{ Hz}$$
$$N = f_{beat} \times \Delta t$$
$$= 3 \text{ Hz} \times 3.0 \text{ s}$$
$$= 3 \text{ s}^{-1} \times 3.0 \text{ s}$$
$$= 9$$

There will be 9 beats heard in 3 s.

26. Frame the Problem
- The trumpet and piano notes are being sounded together.
- Beats are heard, so the frequencies are close.
- The beat frequency gives the difference in frequencies.
- There are two possiblities: the trumpet may be a little above or a little below the piano note.

Identify the Goal
The two possible frequencies, f_{t_1} and f_{t_2}, for the trumpet note

Variables and Constants

Known	Unknown
$f_p = 256$ Hz	f_{t_1}
$N = 10$	f_{t_2}
$\Delta t = 2.0$ s	

Strategy
Find beat frequency from N and Δt.
Trumpet note is either that many Hz above piano note or that many below piano note

$$f_{beat} = \frac{N}{\Delta t}$$
$$= \frac{10}{2.0 \text{ s}}$$
$$= 5.0 \text{ Hz}$$
$$f_{t1} = f_p + f_{beat}$$
$$= 256 \text{ Hz} + 5 \text{ Hz}$$
$$= 261 \text{ Hz}$$
$$f_{t2} = f_p - f_{beat}$$
$$= 256 \text{ Hz} - 5 \text{ Hz}$$
$$= 251 \text{ Hz}$$

The trumpet frequency could be 251 Hz or 261 Hz.

Validate
The possible trumpet frequencies should be 5 Hz on either side of 256 Hz.

27. Frame the Problem
- The piano note and tuning fork are played together; beats are heard.
- The piano note must be a little above or below the fork note.
- Tension on string is slightly increased, so piano note is raised.
- More beats are heard, so the sound of the piano note is moving away from tuning fork frequency.

Identify the Goal
(a) Original frequency of piano note, f_P

Variables and Constants

Known	Unknown
$f_f = 440$ Hz	f_{beat}
$N = 12$	f_P
$\Delta t = 4.0$ s	

Strategy
Find beat frequency from N and Δt
The sound of the piano note must be that many Hz above the sound of the fork note because raising the frequency did not synchronize the two sounds.
$$f_{beat} = \frac{N}{\Delta t}$$
$$= \frac{12}{4.0 \text{ s}}$$
$$= 3.0 \text{ Hz}$$
$$f_P = f_f + f_{beat}$$
$$= 440 \text{ Hz} + 3 \text{ Hz}$$
$$= 443 \text{ Hz}$$

The piano string's original frequency was 443 Hz.
(**Note:** There is a mathematical possibility for the original string to have been 437 Hz. When the tuner tightened the string, he might have pulled the string *past* the correct frequency, all the way to 443.5 Hz. The use of the word "slightly" in this question can rule out this possibility.)

(b) The piano is more out of tune after tightening the string, because more beats were heard per second.

Chapter 9 Review

Answers to Problems for Understanding

40. (a) $v = 331$ m/s $+ 0.59\,T_C$
$= 331$ m/s $+ 0.59\frac{\text{m/s}}{°C}(-40°C)$
$= 307$ m/s

(b) $v = 331$ m/s $+ 0.59\,T_C$
$= 331$ m/s $+ 0.59\frac{\text{m/s}}{°C}(5°C)$
$= 334$ m/s

(c) $v = 331$ m/s $+ 0.59\,T_C$
$= 331$ m/s $+ 0.59\frac{\text{m/s}}{°C}(21°C)$
$= 343$ m/s

(d) $v = 331$ m/s $+ 0.59\,T_C$
$= 331$ m/s $+ 0.59\frac{\text{m/s}}{°C}(35°C)$
$= 352$ m/s

41. (a) 355 m/s $= 331$ m/s $+ 0.59\,T_C$
$0.59\,T_C = 355$ m/s $- 331$ m/s
$= 24$ m/s
$T_C = \frac{24 \text{ m/s}}{0.59 \text{ m/s/}°C}$
$= 41°C$

(b) 344 m/s $= 331$ m/s $+ 0.59\,T_C$
$0.59\,T_C = 344$ m/s $- 331$ m/s
$= 13$ m/s
$T_C = \frac{13 \text{ m/s}}{0.59 \text{ m/s/}°C}$
$= 22°C$

(c) 333 m/s $= 331$ m/s $+ 0.59\,T_C$
$0.59\,T_C = 333$ m/s $- 331$ m/s
$= 2$ m/s
$T_C = \frac{2 \text{ m/s}}{0.59 \text{ m/s/}°C}$
$= 3.4°C$

(d) 318 m/s $= 331$ m/s $+ 0.59\,T_C$
$0.59\,T_C = 318$ m/s $- 331$ m/s
$= -13$ m/s
$T_C = \frac{-13 \text{ m/s}}{0.59 \text{ m/s/}°C}$
$= -22°C$

42.
$v = \frac{2 \times 250 \text{ m}}{1.5 \text{ s}}$
$= 333.3$ m/s
333.3 m/s $= 331$ m/s $+ 0.59\,T_C$
$0.59\,T_C = 333.3$ m/s $- 331$ m/s
$= 2.3$ m/s
$T_C = \frac{2.3 \text{ m/s}}{0.59 \text{ m/s/}°C}$
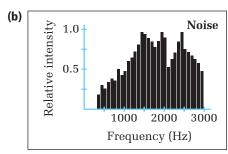$= 3°C$

43. $v = 331 + 0.59\,T_C$

$\quad = 331\text{ m/s} + 0.59\tfrac{\text{m/s}}{°\text{C}}(8°\text{C})$

$\quad = 335.7\text{ m/s}$

$\Delta d = v\Delta t$

$\quad = 335.7\text{ m/s} \times 2.1\text{ s}$

$\quad = 705\text{ m}$

$\quad = 7.0 \times 10^2\text{ m}$

44. (a) $f_\text{beat} = \dfrac{14}{4.0\text{ s}}$

$\qquad = 3.5\text{ Hz}$

Possible frequencies are 436.5 Hz and 443.5 Hz.

(b) He could tighten the string a little and see if the beat frequency reduces. If the beat frequency reduces, he was playing the lower frequency.

45. (a) The oscilloscope pattern would show smooth repetitions, indicating that the notes combined to form a wave that sounds harmonious.

(b)

46. (a) As the pipe is raised, if the open end reaches a length that is $\tfrac{1}{4}$ the wavelength of the tuning fork (or another odd multiple of the wavelength), resonance will occur and the fork's note will be amplified.

(b) An air temperature of 22°C gives a speed of 344 m/s.

$\lambda = \dfrac{v}{f}$

$\quad = \dfrac{344\text{ m/s}}{880\text{ Hz}}$

$\quad = 0.39\text{ m}$

In a closed air column, resonance lengths are odd multiples of a quarter of a wavelength. Therefore, the first four resonance lengths are 9.8 cm, 29 cm, 49 cm, and 68 cm.

47. (a) The interval between resonance lengths in an open air column is half of a wavelength. Thus, the wavelength is $2 \times (57\text{ cm} - 38\text{ cm}) = 38\text{ cm}$.

(b) At 18°C, the speed of sound is 341.6 m/s.

$f = \dfrac{v}{\lambda}$

$\quad = \dfrac{341.6\text{ m/s}}{0.38\text{ m}}$

$\quad = 899\text{ Hz}$

48. If the well was 500 m deep and the stone was not subject to air resistance, the trip down of the rock itself would take about 10 s. If the well was about 140 m deep, the trip down would take about 5.3 s and the time for the sound to return would be about 0.7 s.

49. $\lambda = \frac{v}{f}$

$\quad = \frac{340 \text{ m/s}}{5500 \text{ Hz}}$

$\quad = 0.0618 \text{ m}$

The wavelength of the siren is 0.062 m.

50. $v = 2.4 \times 320 \text{ m/s} \times \frac{3600 \text{ s/h}}{1000 \text{ m/km}}$

$\quad = 2764 \text{ m/km}$

$\quad = 2.7 \times 10^3 \text{ m/km}$

51. Step 1: Find speed of sound at 31°C:

$v = 331 \text{ m/s} + (0.59 \text{ m/s/°C})(31°C)$

$\quad = 349.3 \text{ m/s}$

Step 2: Find distance

$\text{distance} = \frac{0.75 \text{ s}}{2} \times 349.3 \text{ m/s}$

$\quad = 131 \text{ m}$

$\quad = 1.3 \times 10^2 \text{ m}$

52. Time for balloon to fall, assuming constant acceleration of 9.8 m/s², is

$\Delta t = \sqrt{\frac{2\Delta d}{a}}$

$\quad = \sqrt{\frac{2(12.0 \text{ m})}{9.8 \text{ m/s}^2}}$

$\quad = 1.56 \text{ s}$

Person cries out after 1.5 s, therefore sound must reach person in 0.06 s. Assuming the temperature to be 20°C, and thus the speed of sound to be 343 m/s, time for sound to reach person is $\frac{(12.0 \text{ m})}{(343 \text{ m/s})} = 0.0350 \text{ s}$. Therefore warning will reach person in time for person to make an instantaneous jump out of the way.

53. (a) The angle of reflection is 55°.

(b) The angle between the incident ray and the reflected ray is 110°.

54. The angle of reflection is 90° − 34° = 56°, which is the angle of incidence.

55. The angle of incidence is 27°. When the mirror is tilted, the new angle of incidence will be 19°. The angle between the original incident ray, and the reflected ray will be 54°. The angle between the new incident ray, and its reflected ray will be (27° − 8°) × 2 = 38°. The total change in the angle of reflection will be 54° − 38° = 16°.

56. $n_i \sin \theta_i = n_R \sin \theta_R$

$n_R = \frac{n_i \sin \theta_i}{\theta_R} = \frac{1.333 \times \sin 70°}{\sin 40.0°} = 1.95$

57. $n_i \sin \theta_i = n_R \sin \theta_R$

$\sin \theta_R = \frac{n_i \sin \theta_i}{n_R} = \frac{1.51 \times \sin 20.0°}{1.333} = 0.3874$

$\theta_R = \sin^{-1} 0.3874 = 22.8°$

58. The light ray enters the hypotenuse of a right-triangle retroreflector 3.5 cm from the lower corner at an angle of 30° to the normal. Its angle of refraction is 19.2° to the normal. The ray continues to the bottom of the retroreflector where its angle of incidence and reflection is 64.2° to the normal. It then reflects off the side, with angles of incidence and reflection 25.8° to the normal, and travels back to the hypotenuse where its angle of incidence is 19.2° and angle of refraction is 30°. The exiting ray is parallel to the incoming ray.

59. $n = \frac{c}{v}$; $v = \frac{c}{n} = \frac{3.00 \times 10^8 \text{ m/s}}{1.333} = 2.25 \times 10^8 \text{ m/s}$

$v = \frac{\Delta d}{\Delta t}$; $\Delta t = \frac{\Delta d}{v} = \frac{0.54 \text{ m}}{2.25 \times 10^8 \text{ m/s}} = 2.4 \times 10^{-9} \text{ s}$

60. (a) $n_i \sin \theta_i = n_R \sin \theta_R$

$n_R = \frac{n_i \sin \theta_i}{\sin \theta_R} = \frac{1.00 \times \sin 57°}{\sin 44°} = 1.207 = 1.2$

(b) $n_i \sin \theta_i = n_R \sin \theta_R$

$\sin \theta_R = \frac{n_i \sin \theta_i}{n_R} = \frac{1.00 \times \sin 27°}{2.42} = 0.1876$

$\theta_R = \sin^{-1} 0.1876 = 10.81° = 11°$

(c) $n_i \sin \theta_i = n_R \sin \theta_R$

$\sin \theta_i = \frac{n_R \sin \theta_R}{n_i} = \frac{1.33 \times \sin 28°}{1.00} = 0.6244$

$\theta_R = \sin^{-1} 0.6244 = 38.64° = 39°$

61. $n_i \sin \theta_i = n_R \sin \theta_R$

$\sin \theta_i = \frac{n_R \sin \theta_R}{n_i} = \frac{1.362 \times \sin 25°}{1.54} = 0.3738$

$\theta_i = \sin^{-1} 0.3738 = 38.64° = 21.95°$

62. First, determine the index of refraction of the special glass:

$n_i \sin \theta_i = n_R \sin \theta_R$

$n_i = \frac{n_R \sin \theta_R}{\sin \theta_i} = \frac{1.00 \times \sin 90°}{44°} = 1.4396$

Next, determine apply Snell's law again to determine the critical angle:

$n_i \sin \theta_i = n_R \sin \theta_R$

$\sin \theta_i = \frac{n_R \sin \theta_R}{n_i} = \frac{1.333 \times \sin 90°}{1.4396°} = 0.9260$

$\theta_i = \sin^{-1} 0.9260 = 67.81° = 98°$

63. (a) $v = \frac{\Delta d}{\Delta t}$; $\Delta d = v \Delta t = 3.00 \times 10^8 \text{ m/s} \cdot \frac{3 \times 10^{-10} \text{ s}}{2} = \pm 0.045 \text{ m} = \pm 4.5 \text{ cm}$

(b) A laser has a fine, high-intensity, collimated beam that does not spread out as it travels.

64. The wavelength of the blue light was 4.7×10^{-7} m.

$$\lambda \cong \frac{\Delta y d}{x}$$

$$\lambda \cong \frac{(21.1 \times 10^{-3} \text{ m})(1.8 \times 10^{-5} \text{ m})}{0.80 \text{ m}}$$

$$\lambda \cong 4.7 \times 10^{-7} \text{ m}$$

65. The wavelength of the sodium-vapour lamp is 5.89×10^{-7} m or 589 nm.

$$\lambda \cong \frac{\Delta y d}{x}$$

$$\lambda \cong \frac{(0.589 \times 10^{-3} \text{ m})(1.00 \times 10^{-3} \text{ m})}{1.00 \text{ m}}$$

$$\lambda \cong 5.89 \times 10^{-7} \text{ m}$$