Chapter 11

Projectiles and Circular Motion

Practice Problem Solutions

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1. **Conceptualize the Problem**
   - Make a sketch of the problem and superimpose a coordinate system on it. As indicated in the figure in the text, "down" is chosen as *negative;* let the drop point be $x = 0$.
   - Remember that in projectile problems, the *horizontal velocity* and *vertical velocity* are independent: the *horizontal velocity* is constant, while the *vertical velocity* changes because of the *acceleration due to gravity*.
   - The package *initially* has no *vertical velocity* because the plane is flying at a constant altitude. The package falls, from *rest* (in the *y*-direction), with the *acceleration due to gravity*. "Down" was chosen as *negative*, so the *acceleration* of the package is *negative*.
   - The *vertical displacement* that the package falls determines the *time interval* during which it falls, according to the kinematics equations.
   - The package moves with the same *horizontal velocity* as the airplane. The *horizontal displacement* can be found from the *time interval* and the *horizontal velocity*.

   Identify the Goal

   The horizontal distance, d_x, that the package should be released before the drop point

   Identify the Variables

Known	Implied	Unknown
$\Delta y = -785$ m	$v_{iy} = 0.0$ m/s	Δx
$v_x = 53.5$ m/s	$a_y = -9.81$ m/s^2	Δt

   Develop a Strategy

   Find the time interval that the package was falling by using the kinematics equation that relates displacement, initial velocity, acceleration and time interval.

   Calculations

   $$\Delta y = v_{iy}\Delta t + \frac{1}{2}a\Delta t^2$$
   $$\Delta y = \frac{1}{2}a\Delta t^2$$
   $$\Delta t = \sqrt{\frac{2\Delta y}{a}}$$

   Substitute and solve.

   $$\Delta t = \sqrt{\frac{2(-785\text{ m})}{-9.81\text{ m/s}^2}}$$
   $$\Delta t = \pm 12.651\text{ s}$$

   Choose the positive value for time, since negative time has no meaning in this application.

Find the horizontal displacement of the package by using the equation for uniform motion (constant velocity) that relates velocity, displacement and time interval. Solve for displacement.

$$v_x = \frac{\Delta x}{\Delta t}$$
$$\Delta x = v_x \Delta t$$
$$\Delta x = (53.5\ \text{m/s})(12.651\ \text{s})$$
$$\Delta x = 676.8\ \text{m}$$
$$\therefore \Delta x \cong 677\ \text{m}$$

The package should be dropped 677 m before the drop point.

Validate the Solution

The package falls a distance of nearly 800 m in 12.6 s, which seems reasonable. Similarly, an object moving with a velocity of 53.5 m/s will travel several hundred metres in 12.6 s, as is observed here.

2. Conceptualize the Problem

- Make a sketch of the problem and superimpose a coordinate system on it. Let "down" be *negative*.
- The cougar *initially* has no *vertical velocity*. It falls, from *rest* (in the *y*-direction), with the *acceleration due to gravity*. "Down" was chosen as *negative*, so its *acceleration* is *negative*.
- The *vertical displacement* that the cougar falls determines the *time interval* during which it falls, according to the kinematics equations.
- The cougar moves with a constant *horizontal velocity*, which can be found from the *horizontal displacement* and the *time interval*.

Identify the Goal

The horizontal velocity, v_x, of the cougar to land on the rabbit

Identify the Variables

Known	Implied	Unknown
$\Delta y = -3.82$ m	$v_{1y} = 0.0$ m/s	v_x
$\Delta x = 4.12$ m	$a_y = -9.81$ m/s^2	Δt

Develop a Strategy / **Calculations**

Find the time interval that the cougar was falling by using the kinematics equation that relates displacement, initial velocity, acceleration and time interval.

$$\Delta y = v_{1y}\Delta t + \frac{1}{2}a\Delta t^2$$
$$\Delta y = \frac{1}{2}a\Delta t^2$$
$$\Delta t = \sqrt{\frac{2\Delta y}{a}}$$

Substitute and solve.

$$\Delta t = \sqrt{\frac{2(-3.82\ \text{m})}{-9.81\ \text{m/s}^2}}$$
$$\Delta t = \pm 0.8825\ \text{s}$$

Choose the positive value for time, since negative time has no meaning in this application.

Find the horizontal velocity of the cougar by using the equation for uniform motion (constant velocity) that relates velocity, displacement and time interval.

$$v_x = \frac{\Delta x}{\Delta t}$$
$$v_x = \frac{4.12\ \text{m}}{0.8825\ \text{s}}$$
$$v_x = 4.6686\ \text{m/s}$$
$$\therefore v_x \cong 4.67\ \text{m/s}$$

The cougar should jump from the branch with a horizontal velocity of 4.67 m/s.

Validate the Solution

Based on the small distance travelled, the time and horizontal velocity seem reasonable.

3. **Conceptualize the Problem**

- Make a sketch of the problem and superimpose a coordinate system on it. Let "down" be *negative*.
- The skier *initially* has no *vertical velocity*. The skier falls, from *rest* (in the *y*-direction), with the *acceleration due to gravity*. "Down" was chosen as *negative*, so the skier's *acceleration* is *negative*.
- The *vertical displacement* that the skier falls determines the *time interval* during which she falls, according to the kinematics equations.
- The skier moves with a constant *horizontal velocity*. It can be used with the *time interval* to find the *horizontal displacement*.
- The *final velocity* can be found once the v*ertical velocity* is known. The final *vertical velocity*, or *y*-component of the velocity, can be found from the kinematical relation involving the *initial vertical velocity*, the *acceleration*, and the *time interval*.
- The *final velocity* of the skier is then the vector sum of the *horizontal* and *vertical velocities*.

Identify the Goal

The horizontal distance, Δx, that the skier jumped

The skier's velocity, v, when she landed

Identify the Variables

Known	Implied	Unknown
$\Delta y = -78.5\text{ m}$	$v_{iy} = 0.0\text{ m/s}$	Δx
$v_{ix} = 22.4\text{ m/s}$	$a_y = -9.81\text{ m/s}^2$	Δt
		$\vec{v}_f$

Develop a Strategy

Find the time interval that the skier was falling by using the kinematics equation that relates displacement, initial velocity, acceleration and time interval.

Calculations

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a\Delta t^2$$

$$\Delta y = \frac{1}{2}a\Delta t^2$$

$$\Delta t = \sqrt{\frac{2\Delta y}{a}}$$

Substitute and solve.

Choose the positive value for time, since negative time has no meaning in this application.

$$\Delta t = \sqrt{\frac{2(-78.5\text{ m})}{-9.81\text{ m/s}^2}}$$

$$\Delta t = \pm 4.00\text{ s}$$

Find the horizontal distance the skier travels by using the equation for uniform motion (constant velocity) that relates velocity, displacement and time interval.

$$v_x = \frac{\Delta x}{\Delta t}$$

$$\Delta x = v_x\Delta t$$

$$\Delta x = (22.4\text{ m/s})(4.00\text{ s})$$

$$\Delta x = 89.6\text{ m}$$

The skier travelled a horizontal distance of 89.6 m.

Find the vertical component of the final velocity by using the kinematics relation that relates initial velocity, final velocity, acceleration and time.

$$v_{fy} = v_{iy} + a\Delta t$$
$$v_{fy} = 0 + (-9.81\ \text{m/s}^2)(4.00\ \text{s})$$
$$v_{fy} = -39.24\ \text{m/s}$$

Use the Pythagorean theorem to find the magnitude of the resultant velocity.

$$|\vec{v_f}| = \sqrt{(v_{fx})^2 + (v_{fy})^2}$$
$$|\vec{v_f}| = \sqrt{(22.4\ \text{m/s})^2 + (-39.24\ \text{m/s})^2}$$
$$|\vec{v_f}| = 45.183\ \text{m/s}$$
$$|\vec{v_f}| \cong 45.2\ \text{m/s}$$

Use trigonometry to find the angle that the skier made with the horizontal when she struck the ground.

$$\tan\theta = \frac{v_{fy}}{v_x}$$
$$\theta = \tan^{-1}\frac{v_{fy}}{v_x}$$
$$\theta = \tan^{-1}\frac{-39.24\ \text{m/s}}{22.4\ \text{m/s}}$$
$$\theta = -60.3°$$

The skier hits the ground with a velocity of 45.2 m/s at an angle of 60.3° below the horizontal.

Validate the Solution

The skier hits the ground at an angle of 60.3° below the horizontal which seems reasonable for a long ski jump.

4. Conceptualize the Problem

- Make a sketch of the problem and superimpose a coordinate system on it. Because of the phrasing of the question, let "down" be *positive*.
- The arrow *initially* has no *vertical velocity*. The arrow falls, from *rest* (in the *y*-direction), with the *acceleration due to gravity*. "Down" was chosen as *positive*, so the arrow's *acceleration* is *positive*.
- The arrow moves with a constant *horizontal velocity*. This can be used with the *horizontal displacement* to find the *time interval* that the arrow is in the air.
- The *vertical displacement* of the arrow while in the air, is determined by the *time interval* and the *acceleration due to gravity*, according to the kinematics equations.

Identify the Goal

The vertical distance, Δy, that the arrow falls below its point of release

Identify the Variables

Known	Implied	Unknown
$v_x = 70.1$ m/s	$v_{iy} = 0.0$ m/s	Δy
$\Delta x = 12.5$ m	$a_y = +9.81$ m/s^2	Δt

Develop a Strategy

Find the time interval that the arrow was in in the air by using the kinematics equation for uniform (constant) velocity that relates horizontal displacement, horizontal velocity and time interval.

Substitute and solve.

Calculations

$$v_x = \frac{\Delta x}{\Delta t}$$
$$\Delta t = \frac{\Delta x}{v_x}$$
$$\Delta t = \frac{12.5\ \text{m}}{70.1\ \text{m/s}}$$
$$\Delta t = 0.1783\ \text{s}$$

Find the vertical distance the arrow falls by using the kinematics equation that relates initial velocity, time interval and acceleration to vertical displacement.

$$\Delta y = v_{1y}\Delta t + \frac{1}{2}a\Delta t^2$$
$$\Delta y = 0 + \frac{1}{2}(9.81 \text{ m/s}^2)(0.1783 \text{ s})^2$$
$$\Delta y = 0.15596 \text{ m}$$
$$\Delta y \cong 0.156 \text{ m}$$

The arrow fell 0.156 m below its point of release.

Validate the Solution

The arrow has a high horizontal velocity so it's expected that the vertical displacement over the small horizontal displacement will be small.

5. Conceptualize the Problem

- Make a sketch of the problem and superimpose a coordinate system on it. Let "down" be *negative*.
- The golf ball *initially* has no *vertical velocity*. It falls, from *rest* (in the *y*-direction), with the *acceleration due to gravity*. "Down" was chosen as *negative*, so the golf ball's *acceleration* is *negative*.
- The *vertical displacement* that the golf ball falls determines the *time interval* during which it falls, according to the kinematics equations.
- The golf ball moves with a constant *horizontal velocity*. This can be found from the *horizontal displacement* (given) and the *time interval* (found above).

Identify the Goal

The initial (horizontal) velocity, v_x, of the golf ball

Identify the Variables

Known	Implied	Unknown
$\Delta y = 1.22$ m	$v_{1y} = 0.0$ m/s	v_x
$\Delta x = 1.52$ m	$a_y = -9.81$ m/s^2	Δt

Develop a Strategy

Calculations

Find the time interval that the golf ball was falling by using the kinematics equation that relates displacement, initial velocity, acceleration and time interval.

$$\Delta y = v_{1y}\Delta t + \frac{1}{2}a\Delta t^2$$
$$\Delta y = \frac{1}{2}a\Delta t^2$$
$$\Delta t = \sqrt{\frac{2\Delta y}{a}}$$

Substitute and solve.

$$\Delta t = \sqrt{\frac{2(-1.22 \text{ m})}{-9.81 \text{ m/s}^2}}$$
$$\Delta t = \pm 0.4987 \text{ s}$$

Choose the positive value for time, since negative time has no meaning in this application.

Find the initial horizontal velocity of the golf ball from the horizontal displacement and the time interval. Substitute and solve.

$$v_x = \frac{\Delta x}{\Delta t}$$
$$v_x = \frac{1.52 \text{ m}}{0.4987 \text{ s}}$$
$$v_x = 3.0478 \text{ m/s}$$
$$v_x \cong 3.05 \text{ m/s}$$

The initial velocity of the golf ball as it leaves the edge of the table is 3.05 m/s.

Validate the Solution

Considering the time interval and distances involved, the initial velocity of 3.05 m/s seems reasonable.

6. Conceptualize the Problem

- Make a sketch of the problem and superimpose a coordinate system on it. Let "down" be *negative*.
- The baseball *initially* has no *vertical velocity*. It falls, from *rest* (in the *y*-direction), with the *acceleration due to gravity*. "Down" was chosen as *negative*, so the baseball's *acceleration* is *negative*.
- The *vertical displacement* that the baseball falls determines the *time interval* during which it falls, according to the kinematics equations.
- The baseball moves with a constant *horizontal velocity*. This can be used with the *time interval* (found above) to find the *horizontal displacement*.

Identify the Goal

The horizontal displacement, Δx, of the baseball

Identify the Variables

Known	Implied	Unknown
$v_x = 1.0$ m/s	$v_{1y} = 0.0$ m/s	Δx
$\Delta y = 1.5$ m	$a_y = -9.81$ m/s^2	Δt

Develop a Strategy	Calculations
Find the time interval that the baseball was falling by using the kinematics equation that relates displacement, initial velocity, acceleration and time interval.	$\Delta y = v_{1y}\Delta t + \frac{1}{2}a\Delta t^2$ $\Delta y = \frac{1}{2}a\Delta t^2$ $\Delta t = \sqrt{\frac{2\Delta y}{a}}$
Substitute and solve.	$\Delta t = \sqrt{\frac{2(-1.5\text{ m})}{-9.81\text{ m/s}^2}}$
Choose the positive value for time, since negative time has no meaning in this application.	$\Delta t = \pm 0.5530$ s
Find the horizontal displacement of the baseball from the horizontal velocity and the time interval. Substitute and solve.	$v_x = \frac{\Delta x}{\Delta t}$ $\Delta x = v_x\Delta t$ $\Delta x = (1.0\text{ m/s})(0.5530\text{ s})$ $\Delta x = 0.5530$ m $\Delta x \cong 0.55$ m
The horizontal displacement of the baseball is about 0.55 m.	

Validate the Solution

For a ball rolling off a shelf, a horizontal displacement of about 0.55 m seems reasonable.

7. Conceptualize the Problem

- Make a sketch of the problem and superimpose a coordinate system on it. Let "down" be *negative*.
- The stone *initially* has no *vertical velocity*. It falls, from *rest* (in the *y*-direction), with the *acceleration due to gravity*. "Down" was chosen as *negative*, so the stone's *acceleration* is *negative*.

- The *vertical displacement* that the stone falls determines the *time interval* during which it falls, according to the kinematics equations.
- The stone moves with a constant *horizontal velocity*. This can be used with the *time interval* (found above) to find the *horizontal displacement*.

Identify the Goal

The horizontal displacement, Δx, of the baseball

Identify the Variables

Known	Implied	Unknown
$v_x = 22$ m/s	$v_{1y} = 0.0$ m/s	Δx
$\Delta y = 55$ m	$a_y = -9.81$ m/s^2	Δt

Develop a Strategy

Find the time interval that the stone was falling by using the kinematics equation that relates displacement, initial velocity, acceleration and time interval.

Calculations

$$\Delta y = v_{1y}\Delta t + \frac{1}{2}a\Delta t^2$$

$$\Delta y = \frac{1}{2}a\Delta t^2$$

$$\Delta t = \sqrt{\frac{2\Delta y}{a}}$$

Substitute and solve.

$$\Delta t = \sqrt{\frac{2(-55\text{ m})}{-9.81\text{ m/s}^2}}$$

Choose the positive value for time, since negative time has no meaning in this application.

$$\Delta t = \pm 3.3486\text{ s}$$

Find the horizontal displacement of the stone from the horizontal velocity and the time interval.

Substitute and solve.

$$v_x = \frac{\Delta x}{\Delta t}$$

$$\Delta x = v_x\Delta t$$

$$\Delta x = (22\text{ m/s})(3.3486\text{ s})$$

$$\Delta x = 73.6689\text{ m}$$

$$\Delta x \cong 74\text{ m}$$

The horizontal displacement of the stone is about 74 m.

Validate the Solution

The stone is in the air for more than 3 s, and has a high initial velocity, so a horizontal displacement of about 74 m seems reasonable.

8. Conceptualize the Problem

- Make a sketch of the problem and superimpose a coordinate system on it. Let "down" be *negative*.
- The shell starts from rest, *accelerates due to gravity* and strikes the ground with a given *final velocity*. From the appropriate kinematics equation, the *vertical distance* that the shell falls can be determined. This will be the same *vertical displacement* for the bullet.
- Both the bullet and the shell take the same length of *time* to strike the ground. Because down was chosen as negative, both the bullet and shell will have *negative acceleration*.
- The bullet has a constant *horizontal velocity*. This can be used with the *time interval* to find the *horizontal displacement*.
- The *vertical component* of the bullet's velocity changes from zero (rest) to some final value. Because the bullet and the shell have the same vertical *acceleration*, it is expected that the *vertical component* of their final velocities will be the same.

Identify the Goal

(a) The horizontal displacement, Δx, of the bullet

(b) The vertical component of the bullet's final velocity, $v_{\mathrm{fy\_bullet}}$

Identify the Variables

Known	Implied	Unknown
$v_x = 3.00 \times 10^2$ m/s	$v_{\mathrm{iy\_shell}} = 0.0$ m/s	Δx
	$v_{\mathrm{iy\_bullet}} = 0.0$ m/s	Δy
$v_{\mathrm{fy\_shell}} = 5.00$ m/s	$a_{\mathrm{ay}} = -9.81$ m/s^2	Δt
		$v_{\mathrm{fy\_bullet}}$

Develop a Strategy	Calculations
Find the vertical displacement of the shell (which is the same for the bullet) by using the kinematics equation that relates displacement, initial velocity, and acceleration to the final velocity. Substitute and solve.	$v^2_{\mathrm{fy\_shell}} = v^2_{\mathrm{iy\_shell}} + 2a\Delta y$ $v^2_{\mathrm{fy\_shell}} = 2a\Delta y$ $\Delta y = \dfrac{v^2_{\mathrm{fy\_shell}}}{2a}$ $\Delta y = \dfrac{(5.00\ \mathrm{m/s})^2}{2(-9.81\ \mathrm{m/s^2})}$ $\Delta y = -1.2742$ m
Find the time interval for the shell to fall by using the kinematics equation that relates the acceleration and the time interval (and the initial velocity) to the vertical displacement. Solve for the time interval. Substitute and solve.	$\Delta y = v_{\mathrm{iy}}\Delta t + \frac{1}{2}a\Delta t^2$ $\Delta y = \frac{1}{2}a\Delta t^2$ $\Delta t = \sqrt{\dfrac{2\Delta y}{a}}$ $\Delta t = \sqrt{\dfrac{2(-1.2742\ \mathrm{m})}{-9.81\ \mathrm{m/s^2}}}$
Choose the positive value for time, since negative time has no meaning in this application. The time interval is the same for the shell and the bullet.	$\Delta t = \pm 0.50968$ s
Find the horizontal displacement of the bullet from the time interval and the bullet's horizontal velocity.	$v_x = \dfrac{\Delta x}{\Delta t}$ $\Delta x = v_x\Delta t$ $\Delta x = (3.00 \times 10^2\ \mathrm{m/s})(0.50968\ \mathrm{s})$ $\Delta x = 152.9$ m $\Delta x \cong 153$ m
(a) The bullet travels a horizontal distance of approximately 153 m. Find the vertical component of the bullet's final velocity from the kinematics equation that relates initial velocity, acceleration and time interval to the final velocity.	$v_{\mathrm{fy}} = v_{\mathrm{iy}} + a\Delta t$ $v_{\mathrm{fy}} = 0 + (-9.81\ \mathrm{m/s^2})(0.50968\ \mathrm{s})$ $v_{\mathrm{fy}} = -5.00$ m/s
(b) The vertical component of the bullet's final velocity is −5.00 m/s.	

Validate the Solution

The bullet has an initial velocity of 300 m/s and is in the air for about half a second, so the horizontal distance of 153 m seems reasonable. Because both the bullet and shell are accelerating due to gravity, they will have the same vertical component of the final velocity. The calculated value for the bullet agrees with the given value for the shell.

Practice Problem Solutions

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9. Conceptualize the Problem

- Make a sketch of the problem and superimpose a *coordinate system* on it. Let "down" be *negative*.
- The stone has a *positive initial velocity* in both the *vertical* and *horizontal* directions. It will rise and then fall according to the kinematics equations.
- The *vertical acceleration* of the stone is *negative* and has the *magnitude* of the *acceleration due to gravity*.
- The stone rises to a maximum height, where its vertical velocity is zero, and then falls to the ground. The vertical motion determines the *time* of travel.
- Motion in the *horizontal* direction is *uniform*; that is, it has a *constant velocity*.
- The horizontal *displacement* of the ball depends on the *horizontal component* of the *initial velocity* and on the *time* of travel.

Identify the Goal

The horizontal distance, Δx, the stone lands from the base of the cliff

Identify the Variables

Known	Implied	Unknown
$\lvert\vec{v}_i\rvert = 21$ m/s	$a_y = -9.81$ m/s^2	Δx
$\theta_i = 35°$		Δt
$\Delta y = -60.0$ m		

Develop a Strategy

Find the horizontal and vertical components of the initial velocity.

Calculations

$v_{ix} = \lvert\vec{v}_i\rvert \cos\theta$ $\quad$ $v_{ix} = \lvert\vec{v}_i\rvert \sin\theta$

$v_{ix} = (21 \text{ m/s})(\cos 35°)$ $\quad$ $v_{ix} = (21 \text{ m/s})(\sin 35°)$

$v_{ix} = 17.202$ m/s $\quad$ $v_{ix} = 12.045$ m/s

Find the time interval that the stone was falling by using the kinematics equation that relates displacement, initial velocity, acceleration and time interval.

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a\Delta t^2$$

$$-60.0 \text{ m} = (12.045 \text{ m/s})\Delta t - \frac{1}{2}(-9.81 \text{ m/s}^2)\Delta t^2$$

You cannot solve directly for the time interval, because you have a quadratic equation.

$$4.905\Delta t^2 - 12.045\Delta t - 60.0 = 0$$

Rearrange into the general form of a quadratic equation and solve using the quadratic formula

$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Choose the positive solution.

$$\Delta t = \frac{12.045 \pm \sqrt{(12.045)^2 - 4(4.905)(-60.0)}}{2(4.905)}$$

$\Delta t = 4.9345$ s or -2.4792 s

$\Delta t \cong 4.93$ s

Use the time interval with the equation for uniform (constant) velocity to find the horizontal distance of the stone.

$$v_x = \frac{\Delta x}{\Delta t}$$

$\Delta x = v_x\Delta t$

$\Delta x = (17.202 \text{ m/s})(4.9345 \text{ s})$

$\Delta x = 84.88$ m

$\Delta x \cong 85$ m

The horizontal distance that the stone travels is about 85 m.

Validate the Solution

All the units worked out properly. The quadratic formula provided a solution, indicating that all the terms in the equation had the correct sign (a common error is to have a wrong sign in the quadratic equation and thus having to take the square root of a negative number). The stone was in the air for nearly 5 seconds and had a horizontal velocity of close to 20 m/s, so the horizontal distance should be close to 100 m. The result of 85 m seems reasonable.

10. Conceptualize the Problem

- Make a sketch of the problem and superimpose a *coordinate system* on it. Let "down" be *negative*.
- The baseball has a *positive initial velocity* in both the *vertical* and *horizontal* directions. It will rise and then fall according to the kinematics equations.
- The *vertical acceleration* of the baseball is *negative* and has the *magnitude* of the *acceleration due to gravity*.
- The baseball rises to a maximum height, where its vertical velocity is zero, and then falls to the ground. The vertical motion determines the *time* of travel.
- Motion in the *horizontal* direction is *uniform*; that is, it has a *constant velocity*. However, this information is not needed in this problem.
- The *vertical displacement* of the ball depends on the *vertical component* of the *initial velocity*, the *acceleration* and the *time* of travel.

Identify the Goal

The vertical distance, Δy, the fan is above the field when the ball is caught

Identify the Variables

Known	Implied	Unknown
$\lvert\vec{v}_i\rvert = 41$ m/s	$a_y = -9.81$ m/s^2	Δy
$\theta_i = 47°$		Δt
$v_{fy} = -11$ m/s		

Develop a Strategy	Calculations
Find the horizontal and vertical components of the initial velocity.	$v_{ix} = \lvert\vec{v}_i\rvert\cos\theta$ $\quad v_{iy} = \lvert\vec{v}_i\rvert\sin\theta$ $v_{ix} = (41\text{ m/s})(\cos 47°)$ $\quad v_{iy} = (41\text{ m/s})(\sin 47°)$ $v_{ix} = 27.96\text{ m/s}$ $\quad v_{iy} = 29.98\text{ m/s}$
Find the time interval that the baseball was in the air by using the kinematics equation that relates initial velocity, acceleration and time interval to final velocity. Rearrange to isolate the time interval. Substitute and solve.	$v_{fy} = v_{iy} + a\Delta t$ $\Delta t = \frac{v_{fy} - v_{iy}}{a}$ $\Delta t = \frac{-11\text{ m/s} - 29.98\text{ m/s}}{9.81\text{ m/s}^2}$ $\Delta t = 4.178\text{ s}$
To find the vertical distance of the fan above the field, use the kinematics equation that relates time interval, acceleration and initial velocity to the displacement.	$\Delta y = v_{iy}\Delta t + \frac{1}{2}a\Delta t^2$ $\Delta y = (29.98\text{ m/s})(4.178\text{ s}) + \frac{1}{2}(-9.81\text{ m/s}^2)(4.178)^2$ $\Delta y = 39.64\text{ m}$ $\Delta y \cong 4.0 \times 10^1\text{ m}$
The vertical distance of the fan above the field is about 40 m.	

Validate the Solution

All the units worked out properly. The final result of 40 m above the field is reasonable if the fan is sitting in the upper deck of the stadium.

11. Conceptualize the Problem

- Make a sketch of the problem and superimpose a *coordinate system* on it. Let "down" be *negative*.
- The baseball has a *positive initial velocity* in both the *vertical* and *horizontal* directions. It will rise and then fall according to the kinematics equations.
- The *vertical acceleration* of the baseball is *negative* and has the *magnitude* of the *acceleration due to gravity*.
- The baseball rises to a maximum height, where its vertical velocity is zero, and then falls to the ground. The vertical motion determines the *time* of travel.
- Motion in the *horizontal* direction is *uniform*; that is, it has a *constant velocity*.
- The *final velocity* of the baseball will have both a *vertical* and *horizontal* component.
- The *vertical component* is related to the *time interval*, the *acceleration* and the *initial (vertical) velocity*.

Identify the Goal

The final velocity, v_f, of the baseball before it hits the ground

Identify the Variables

Known	Implied	Unknown
$\lvert\vec{v}_i\rvert = 15\ \text{m/s}$	$a_y = -9.81\ \text{m/s}^2$	$\vec{v}_f$
$\theta_i = 42°$	$v_{fx} = v_{ix}$	Δt

Develop a Strategy

Find the horizontal and vertical components of the initial velocity.

Calculations

$v_{ix} = \lvert\vec{v}_i\rvert \cos\theta$ $\quad$ $v_{iy} = \lvert\vec{v}_i\rvert \sin\theta$

$v_{ix} = (15\ \text{m/s})(\cos 42°)$ $\quad$ $v_{iy} = (15\ \text{m/s})(\sin 42°)$

$v_{ix} = 11.15\ \text{m/s}$ $\quad$ $v_{iy} = 10.04\ \text{m/s}$

Find the time interval that the baseball was in the air by using the kinematic equation that relates displacement, initial velocity, acceleration and time interval.

$$\Delta y = v_{iy}\Delta t + \frac{1}{2}a\Delta t^2$$

$$-5.3\ \text{m} = (10.04\ \text{m/s})\Delta t + \frac{1}{2}(-9.81\ \text{m/s}^2)\Delta t^2$$

You cannot solve directly for the time interval, because you have a quadratic equation.

$$4.905\Delta t^2 - 10.04\Delta t - 5.3 = 0$$

Rearrange into the general form of a quadratic equation and solve using the quadratic formula

$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$\Delta t = \frac{10.04 \pm \sqrt{(10.04)^2 - 4(4.905)(-5.3)}}{2(4.905)}$$

$$\Delta t = 2.4818\ \text{s or } -0.4358\text{s}$$

$$\Delta t \cong 2.48\ \text{s}$$

Choose the positive solution because a negative time interval doesn't have physical meaning.

Find the vertical component of the final velocity by using the kinematic equation that relates time interval, acceleration and initial velocity to the final velocity.

$$v_{fy} = v_{iy} + a\Delta t$$

$$v_{fy} = (10.04\ \text{m/s}) + (-9.81\ \text{m/s}^2)(2.4818\ \text{s})$$

$$v_{fy} = -14.31\ \text{m/s}$$

Use the Pythagorean theorem to find the magnitude of the resultant velocity.

$$\lvert\vec{v}_f\rvert = \sqrt{(v_{fx})^2 + (v_{fy})^2}$$

$$\lvert\vec{v}_f\rvert = \sqrt{(11.15\ \text{m/s})^2 + (-14.31\ \text{m/s})^2}$$

$$\lvert\vec{v}_f\rvert = 18.14\ \text{m/s}$$

$$\lvert\vec{v}_f\rvert \cong 18\ \text{m/s}$$

Use trigonometry to find the angle that the baseball made with the horizontal when it struck the ground.

$$\tan\theta = \frac{v_{fy}}{v_{fx}}$$
$$\theta = \tan^{-1}\frac{v_{fy}}{v_{fx}}$$
$$\theta = \tan^{-1}\frac{-14.31\ \text{m/s}}{11.15\ \text{m/s}}$$
$$\theta = -52.08°$$
$$\theta \cong -52°$$

The baseball strikes the ground with a final velocity of about −18 m/s at an angle of 52° below the horizontal.

Validate the Solution
All the units worked out properly. The time of flight (2.5 s) and final velocity (18 m/s) both seem reasonable.

12. Conceptualize the Problem
- The trajectory of the insect's flight is expected to be a *symmetrical parabola.*
- The equations developed for symmetrical trajectories can be used.

Identify the Goal
The initial velocity, v_i, of the insect

Identify the Variables

Known	Implied	Unknown
$\Delta x = 75$ cm	$a_y = -9.81\ \text{m/s}^2$	$\vec{v}_i$
$\theta_i = 55°$		

Develop a Strategy

Use the equation for the range of a symmetrical trajectory, which relates the angle of the initial velocity, the initial velocity and the acceleration to the horizontal distance.

Calculations

$$\Delta x = \frac{v_i^2 \sin 2\theta}{g}$$
$$v_i^2 = \frac{g\Delta x}{\sin 2\theta}$$
$$v_i = \sqrt{\frac{(9.81\ \text{m/s}^2)(0.75\ \text{m})}{\sin(2 \times 55°)}}$$
$$v_i = 2.7981\ \text{m/s}$$
$$v_i \cong 2.8\ \text{m/s}$$

The initial velocity of the insect is about 2.8 m/s.

Validate the Solution
All the units worked out properly. A launch velocity of 2.8 m/s for an insect seems reasonable.

Practice Problem Solutions

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13. Conceptualize the Problem
- The trajectory of the stunt person's flight will be a *symmetrical parabola.*
- The equations developed for symmetrical trajectories can be used.

Identify the Goal
(a) The position of the safety net, i.e., the horizontal range, *R*, of the stunt person

(b) The maximum height, H

(c) The time of flight, T

Identify the Variables

Known	Implied	Unknown
$v_i = 24.8$ m/s	$g = 9.81$ m/s^2	R
$\theta = 55°$		H
		T

Develop a Strategy

Use the equation for the range of a symmetrical trajectory, which relates the angle of the initial velocity, the initial velocity and the acceleration to the horizontal range.

Calculations

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

$$R = \frac{(24.8 \text{ m/s})^2 \sin(2 \times 55°)}{(9.81 \text{ m/s}^2)}$$

$$R = 58.914 \text{ m}$$

$$R \cong 58.9 \text{ m}$$

(a) The horizontal range of the stunt person is about 58.9 m. This is where the net should be placed.

Use the equation for the maximum height of a symmetrical trajectory.

$$H = \frac{v_i^2 \sin^2 \theta}{2g}$$

$$H = \frac{(24.8 \text{ m/s})^2 (\sin 55°)^2}{2(9.81 \text{ m/s}^2)}$$

$$H = 21.034 \text{ m}$$

$$H \cong 21.0 \text{ m}$$

(b) The maximum height of the stunt person is about 21.0 m.

Use the equation for time of flight of a symmetrical trajectory.

$$T = \frac{2v_i \sin \theta}{g}$$

$$T = \frac{2(24.8 \text{ m/s}) \sin(55°)}{(9.81 \text{ m/s}^2)}$$

$$T = 4.1417 \text{ s}$$

$$T \cong 4.14 \text{ s}$$

(c) The time of flight of the stunt person is about 4.14 s.

Validate the Solution

All the units worked out properly. The stunt would take place within a circus tent, which can be very large. Values of 60 m for the range and 21 m for the height seem reasonable.

14. Conceptualize the Problem

- The trajectory of the stone will be a *symmetrical parabola*.
- The equations developed for symmetrical trajectories can be used.

Identify the Goal

(a) The launch angle, θ, of the stone

(b) The maximum height, H, of the stone

(c) The time of flight, T, of the stone

Identify the Variables

Known	Implied	Unknown
$v_i = 12.5$ m/s	$g = 9.81$ m/s^2	θ
$R = 14.6$ m		H
		T

Develop a Strategy

Use the equation for the range of a symmetrical trajectory, which relates the angle of the initial velocity, the initial velocity and the acceleration to the horizontal range.
Rearrange to find the angle.
Substitute and solve.

Calculations

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

$$\sin 2\theta = \frac{gR}{v_i^2}$$

$$2\theta = \sin^{-1} \frac{(9.81 \text{ m/s}^2)(14.6 \text{ m})}{(12.5 \text{ m/s})^2}$$

$$2\theta = 66.441°$$

$$\theta = \frac{66.441°}{2}$$

$$\theta = 33.22°$$

$$\theta \cong 33.2°$$

(a) The launch angle of the sling shot must be about 33.2° to hit the target.

Use the equation for the maximum height of a symmetrical trajectory.

$$H = \frac{v_i^2 \sin^2 \theta}{2g}$$

$$H = \frac{(12.5 \text{ m/s})^2(\sin 33.22°)^2}{2(9.81 \text{ m/s}^2)}$$

$$H = 2.390 \text{ m}$$

$$H \cong 2.39 \text{ m}$$

(b) The maximum height of the stunt person is about 21.0 m.

Use the equation for time of flight of a symmetrical trajectory.

$$T = \frac{2v_i \sin \theta}{g}$$

$$T = \frac{2(12.5 \text{ m/s}) \sin(33.22°)}{(9.81 \text{ m/s}^2)}$$

$$T = 1.396 \text{ s}$$

$$T \cong 1.40 \text{ s}$$

(c) The time of flight of the stunt person is about 1.40 s.

Validate the Solution

All the units worked out properly. These values, for a slingshot, all seem reasonable.

Practice Problem Solutions

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15. Conceptualize the Problem

- Make a sketch of the motion of the ball on the string and the forces acting on it.
- The *centripetal force* of the string causes the ball to *accelerate* in a horizontal circle.
- The *centripetal force* is related to the *speed;* thus, it can be used to find the *maximum speed* that the string can have without breaking.

Identify the Goal

The maximum speed, v, of the ball on the string

Identify the Variables

Known	Implied	Unknown
$m = 155$ g		v
$r = 1.65$ m		
$F_T = 208$ N		

Develop a Strategy

Use the formula for centripetal force to find the maximum speed that the ball can have.

Calculations

$$F_T = F_{max} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{rF_{max}}{m}}$$

$$v = \sqrt{\frac{(1.65 \text{ m})(208 \text{ N})}{155 \text{ g} \times \frac{1 \text{ kg}}{1000 \text{ g}}}}$$

$$v = 47.055 \text{ m/s}$$

$$v = 47.0 \text{ m/s}$$

The speed of the ball on the string must be less than 47.0 m/s.

Validate the Solution

The units worked out properly for the velocity. The ball has a small mass and the string is less than 2 m long. A speed of 47.0 m/s is quite fast (about 170 km/h) but seems reasonable considering the values of the ball's mass and the length of the string.

16. Conceptualize the Problem

- Make a sketch of the motion of the electron orbiting the hydrogen nucleus and write down the given information.
- Because the electron's *speed, mass and radius* are all given, there is enough information to calculate the *centripetal force* directly.

Identify the Goal

The centripetal force, F, acting on the electron

Identify the Variables

Known	Implied	Unknown
$m = 9.11 \times 10^{-31}$ kg		F_c
$r = 5.3 \times 10^{-11}$ m		
$v = 2.2 \times 10^6$ m/s		

Develop a Strategy

Calculate the centripetal force from the formula for centripetal force.

Calculations

$$F_c = \frac{mv^2}{r}$$

$$F_c = \frac{(9.11 \times 10^{-31} \text{ kg})(2.2 \times 10^6 \text{ m/s})^2}{5.3 \times 10^{-11} \text{ m}}$$

$$F_c = 8.3193 \times 10^{-8} \text{ N}$$

$$F_c \cong 8.3 \times 10^{-8} \text{ N}$$

The centripetal force of the nucleus on the electron is about 8.3×10^{-8} N. It is supplied by the electromagnetic attraction of the positively charged nucleus to the negatively charged electron.

Validate the Solution

This is a straightforward substitution problem. The units work out to be newtons, as required.

17. Conceptualize the Problem

- Draw *free-body diagrams* of the stone at the top and bottom of the vertical circle. Use a coordinate system where the *downward* direction is *negative*.
- At the *top* of the circle, both *tension* and the *force of gravity* are acting *toward the centre* of the circle.
- At the *bottom* of the circle, the *force of gravity* is in the *opposite* direction to the *tension*.
- The string is most likely to break at the bottom of the circle, so find the maximum speed at this point.

Identify the Goal

(a) The tension in the string, F_T, at the top of the revolution

(b) The tension in the string, F_T, at the bottom of the revolution

(c) The maximum speed, v_{max}, the stone can have if the string will break when the tension reaches 33.7 N

Identify the Variables

Known	Implied	Unknown
$m = 284$ g		F_T
$r = 0.850$ m		v_{max}
$v = 12.4$ m/s		

Develop a Strategy

Set the centripetal force equal to the vector sum of the force of tension in the string and the gravitational force when the stone is at the top of the circle.

Calculations

$$-F_T - mg = \frac{mv^2}{r}$$

$$F_T = mg - \frac{mv^2}{r}$$

$$F_T = \left(2.84\text{ g} \times \frac{1.00\text{ kg}}{1000\text{ g}}\right)(9.81\text{ m/s}^2) - \frac{\left(284\text{ g} \times \frac{1.00\text{ kg}}{1000\text{ g}}\right)(12.4\text{ m/s})^2}{(0.850\text{ m})}$$

$$F_T = -48.5979\text{ N}$$

$$F_T \cong -48.6\text{ N}$$

(a) The tension in the string when the stone is at the top is about 48.6 N.

Set the centripetal force equal to the vector sum of the force of tension in the string and the gravitational force when the stone is at the bottom of the circle.

$$F_T - mg = \frac{mv^2}{r}$$

$$F_T = \frac{mv^2}{r} + mg$$

$$F_T = \frac{\left(284\text{ g} \times \frac{1\text{ kg}}{1000\text{ g}}\right)(12.4\text{ m/s})^2}{(0.850\text{ m})} + (284\text{ g})\left(\frac{1\text{ kg}}{1000\text{ g}}\right)(9.81\text{ m/s}^2)$$

$$F_T = 54.16\text{ N}$$

$$F_T \cong 54.2\text{ N}$$

(b) The tension in the string when the stone is at the bottom is about 54.2 N.

The string is most likely to break when the stone is at the bottom of its path. Find the maximum velocity from the vector sum of the forces in (b).

$$F_{Tmax} - mg = \frac{mv_{max}^2}{r}$$

$$v_{max}^2 = r\left(\frac{F_{Tmax} - mg}{m}\right)$$

$$v_{max} = \sqrt{(0.850\text{ m})\left(\frac{33.7\text{ N} - (0.284\text{ kg})(9.81\text{ m/s}^2)}{0.284\text{ kg}}\right)}$$

$$v_{max} = 9.6189\text{ m/s}$$

$$v_{max} \cong 9.62\text{ m/s}$$

(c) The maximum speed of the stone for the given tension is about 9.62 m/s.

Validate the Solution

The weight of the stone is 2.79 N. As the stone travels in the circle, because the tension force has different directions at the top and bottom of the circle (in the same and opposite direction to the weight, respectively), it is expected that the tension at the two places will differ by twice the weight, which it does. The units worked out properly in each case. For part (c), the maximum tension is less than the tension in (a) and (b), where the velocity is higher, and it is expected that the maximum velocity for this case will be less than that in (a) and (b), and it is.

18. Conceptualize the Problem

- Make a sketch of the motion of the car and the forces acting on it.
- The *force of friction* must provide a sufficient *centripetal force* to cause the car to follow the curved road.
- The magnitude of *force* required to keep the car on the road depends on the *velocity* of the car, its *mass*, and the *radius of curvature* of the road.
- This is a straightforward substitution problem.

Identify the Goal

The required frictional force, F_f, between the tires and the road to safely make the turn

Identify the Variables

Known	Implied	Unknown
$m = 1654$ kg		F_f
$r = 129$ m		
$v = 77$ km/h		

Develop a Strategy

The frictional force must provide the centripetal force. Calculate the force from the formula for centripetal force.

Calculations

$$F_f = \frac{mv^2}{r}$$

$$F_f = \frac{(1654\ \text{kg})\left(77\ \frac{\text{km}}{\text{h}} \times \frac{1\ \text{h}}{3600\ \text{s}} \times \frac{1000\ \text{m}}{1\ \text{km}}\right)^2}{129\ \text{m}}$$

$$F_f = 5866\ \text{N}$$

$$F_f \cong 5.9 \times 10^3\ \text{N}$$

The frictional force required to keep the car turning is about 5.9×10^3 N.

Validate the Solution

The units work out to be $\frac{\text{kg}\cdot\text{m}}{\text{s}^2} = \text{N}$, as required.

19. Conceptualize the Problem

- Make a sketch of the motion of the car and the forces acting on it.
- The *force of friction* must provide a sufficient *centripetal force* to cause the car to follow the curved road.
- The magnitude of *force* required to keep the car on the road depends on the *velocity* of the car, its *mass*, and the *radius of curvature* of the road.
- The given *force of friction* and *velocity* apply to the *maximum radius of curvature* that the car can have without skidding.

Identify the Goal

The radius of curvature, *r*, the car has on the turn before it begins to skid

Identify the Variables

Known	Implied	Unknown
$m = 2545$ kg		r
$v = 24$ m/s		
$F_f = 1.75 \times 10^4$ N		

Develop a Strategy

The frictional force must provide the centripetal force. Calculate the radius of curvature from the formula for centripetal force.

Calculations

$$F_c = F_f = \frac{mv^2}{r}$$

$$r = \frac{mv^2}{F_f} = \frac{(2545\ \text{kg})(24\ \text{m/s})^2}{1.75 \times 10^4\ \text{N}}$$

$$r = 83.767\ \text{m}$$

$$r \cong 84\ \text{m}$$

The radius of curvature the car has before skidding is about 84 m.

Validate the Solution

The units work out to be metres, as required.

Practice Problem Solutions

Student Textbook pages 566–567

20. Conceptualize the Problem

- Make a sketch of the motion of the car and the forces acting on it.
- The magnitude of *centripetal force* required to keep the car on the road depends on the *velocity* of the car, its *mass*, and the *radius of curvature* of the road.
- There is enough information to solve the problem in the formula for the *centripetal force*.

Identify the Goal

The radius of curvature, r, of the road

Identify the Variables

Known	Implied	Unknown
$m = 1225$ kg		r
$v = 72.5$ km/h		
$F_c = 4825$ N		

Develop a Strategy

Calculate the radius of curvature from the formula for centripetal force.

Calculations

$$F_c = \frac{mv^2}{r}$$

$$r = \frac{mv^2}{F_c} = \frac{(1225\ \text{kg})\left(72.5\ \frac{\text{km}}{\text{h}} \times \frac{1\ \text{h}}{3600\ \text{s}} \times \frac{1000\ \text{m}}{1\ \text{km}}\right)^2}{4825\ \text{N}}$$

$$r = 102.97\ \text{m}$$

$$r \cong 103\ \text{m}$$

The radius of curvature of the road is about 103 m.

Validate the Solution

The units work out to be metres, as required.

21. Conceptualize the Problem

- The *normal force* of a *banked curve* provides a *centripetal force* to help cars turn without requiring an excessive amount of friction.

- For a given *radius of curvature* and *angle of breaking*, there is *one speed* at which the normal force provides precisely the amount of centripetal force that is needed.

Identify the Goal
The speed, v, for which the normal force provides exactly the required amount of centripetal force for driving around the curve

Identify the Variables

Known	Implied	Unknown
$r = 65$ m	$g = 9.81$ m/s^2	v
$\theta = 15°$		

Develop a Strategy

Write the equation that relates angle of banking, speed, and radius of curvature, and solve for speed, v.

Calculations

$$\tan\theta = \frac{v^2}{rg}$$
$$v^2 = rg\tan\theta$$
$$v = \sqrt{rg\tan\theta}$$
$$v = \sqrt{(65\text{ m})(9.81\text{ m/s}^2)(\tan 15°)}$$
$$v = 13.07\text{ m/s}$$
$$v \cong 13\text{ m/s}$$

The car should travel at a (maximum) speed of about 13 m/s in order to not skid off the road.

Validate the Solution
The units work out to be m/s, as required. The speed of 13 m/s is about 47 km/h, which is a reasonable speed for a highway exit ramp.

22. Conceptualize the Problem
- The *normal force* of a *banked curve* provides a *centripetal force* to help cars turn without requiring an excessive amount of friction.
- For a given *radius of curvature* and *angle of breaking*, there is *one speed* at which the normal force provides precisely the amount of centripetal force that is needed.

Identify the Goal
The speed, v, for which the normal force provides exactly the required amount of centripetal force for driving around the curve without skidding

Identify the Variables

Known	Implied	Unknown
$r = 175$ m	$g = 9.81$ m/s^2	v
$\theta = 12°$		

Develop a Strategy

Write the equation that relates angle of banking, speed, and radius of curvature, and solve for speed, v.

Calculations

$$\tan\theta = \frac{v^2}{rg}$$
$$v^2 = rg\tan\theta$$
$$v = \sqrt{rg\tan\theta}$$
$$v = \sqrt{(175\text{ m})(9.81\text{ m/s}^2)(\tan 12°)}$$
$$v = 19.10\text{ m/s}$$
$$v \cong 19.1\text{ m/s}$$

The car should travel at a (maximum) speed of about 19.1 m/s in order to not skid off the road.

Validate the Solution
The units work out to be m/s, as required. The speed of 19.1 m/s is about 69 km/h, which is a high speed for an icy curve, but still within reason.

23. Conceptualize the Problem
- The *normal force* of a *banked curve* provides a *centripetal force* to help cars turn without requiring an excessive amount of friction.
- For a given *radius of curvature* and *angle of breaking*, there is *one speed* at which the normal force provides precisely the amount of centripetal force that is needed.
- The formula for banked turns contains enough information to solve the problem.

Identify the Goal
The angle, θ, that the road should be banked to accommodate cars travelling at 85 km/h

Identify the Variables

Known	Implied	Unknown
$r = 155$ m	$g = 9.81$ m/s^2	θ
$v = 85$ km/h		

Develop a Strategy

Write the equation that relates angle of banking, speed, and radius of curvature, and solve for speed, v.

Calculations

$$\tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left(\frac{\left(85\ \frac{\text{km}}{\text{h}} \times \frac{1\ \text{h}}{3600\ \text{s}} \times \frac{1000\ \text{m}}{1\ \text{km}}\right)^2}{(155\ \text{m})(9.81\ \text{m/s}^2)}\right)$$

$$\theta = 20.135^\circ$$

$$\theta \cong 20.1^\circ$$

The curve should be banked at about 20.1°

Validate the Solution
The exit speed of the cars is quite high, so it's expected that the angle that the ramp should be curved will be higher than in previous problems, and it is.

Chapter 11 Review

Answers to Problems for Understanding

Student Textbook pages 570–571

15. The most efficient way to find the horizontal distance required for part (a) is to find the time of flight first and use it to find the distance. Therefore, it is best to solve part (b), time of flight, first.

(b) The rock will be in the air for 3.7 s. (A negative time has no meaning in this application.)

$$\Delta y = v_{1y}\Delta t + \frac{1}{2}a_y\Delta t^2$$
$$\Delta y = 0\Delta t + \frac{1}{2}(-g)\Delta t^2$$
$$\Delta t = \sqrt{\frac{2\Delta y}{-g}}$$
$$\Delta t = \pm\sqrt{\frac{2(-68\ \cancel{m})}{-9.81\ \frac{\cancel{m}}{s^2}}}$$
$$\Delta t \cong 3.7\ s$$

(a) The horizontal distance travelled in this time is 3.1×10^1 m.

$$\Delta x = v_x\Delta t$$
$$\Delta x = \left(8.0\ \frac{m}{\cancel{s}}\right)(3.723\ \cancel{s})$$
$$\Delta x \cong 3.0 \times 10^1\ m$$

16. Solving for the time in the air gives 5.5 s.

$$\Delta y = v_{1y}\Delta t + \frac{1}{2}a_y\Delta t^2$$
$$\Delta y = 0\Delta t + \frac{1}{2}(-g)\Delta t^2$$
$$\Delta t = \sqrt{\frac{2\Delta y}{-g}}$$
$$\Delta t = \pm\sqrt{\frac{2(-150\ \cancel{m})}{-9.81\ \frac{\cancel{m}}{s^2}}}$$
$$\Delta t = 5.5\ s$$

In this time, the package travels a horizontal distance of 2.7×10^2 m[N].

$$\Delta x = v_x\Delta t$$
$$\Delta x = \left(175\ \frac{\cancel{km}}{\cancel{h}}\right)\left(\frac{1000\ m}{\cancel{km}}\right)\left(\frac{\cancel{h}}{3600\ \cancel{s}}\right)(5.5\ \cancel{s})$$
$$\Delta x \cong 2.7 \times 10^2\ m$$

17. The horizontal and vertical components of the initial velocity are

$$v_x = v_o \cos\theta \qquad \text{and} \qquad v_{1y} = v_o \sin\theta$$
$$v_x = \left(18\ \frac{m}{s}\right)(\cos 24°) \qquad v_{1y} = \left(18\ \frac{m}{s}\right)(\sin 24°)$$
$$v_x = 16.4\ \frac{m}{s} \qquad v_{1y} = 7.32\ \frac{m}{s}$$

(a) The time of flight is 2.1 s. The negative value for time has no meaning.

$$\Delta y = v_{1y}\Delta t + \frac{1}{2}a_y\Delta t^2$$
$$\frac{1}{2}a_y\Delta t^2 + v_{1y}\Delta t - \Delta y = 0$$
$$\Delta t = \frac{-v_{1y} \pm \sqrt{{v_{1y}}^2 - 4\left(\frac{1}{2}a_y\right)(-\Delta y)}}{2\left(\frac{1}{2}a_y\right)}$$
$$\Delta t = -\frac{-7.32\ \frac{m}{s} \pm \sqrt{\left(7.32\ \frac{m}{s}\right)^2 - 4\left[\frac{1}{2}\left(-9.81\ \frac{m}{s^2}\right)\right][-(-5.8\ m)]}}{2\left[\frac{1}{2}\left(-9.81\ \frac{m}{s^2}\right)\right]}$$
$$\Delta t \cong 2.1\ s \text{ or } -0.575\ s$$

(b) The horizontal distance travelled in 2.06 s is 34 m.

$$\Delta x = v_x \Delta t$$
$$\Delta x = \left(16.4\ \tfrac{\text{m}}{\text{s}}\right)(2.06\ \text{s})$$
$$\Delta x \cong 34\ \text{m}$$

(c) The maximum height is 2.7 m above the roof or 8.5 m above the ground.

$$H = \frac{v_i{}^2\ \sin^2\ \theta}{2g}$$
$$H = \frac{\left(18\ \tfrac{\text{m}}{\text{s}}\right)^2(\sin^2\ 24°)}{2\left(9.81\ \tfrac{\text{m}}{\text{s}^2}\right)}$$
$$H \cong 2.7\ \text{m}$$

(d) The velocity of the ball at 2.0 m above the roof is 17 m/s at 13° counterclockwise from the positive *x*-axis. The horizontal component of the velocity is constant, $v_x = 16.4$ m/s, while the vertical component changes. At a height of 2.0 m, the vertical velocity is 3.8 m/s.

$$v_{fy}{}^2 = v_{iy}{}^2 + 2a_y\Delta y$$
$$v_{fy} = \sqrt{2a_y\Delta y + v_{iy}{}^2}$$
$$v_{fy} = \sqrt{2\left(-9.81\ \tfrac{\text{m}}{\text{s}^2}\right)(2.0\ \text{m}) + \left(7.32\ \tfrac{\text{m}}{\text{s}}\right)^2}$$
$$v_{fy} = 3.8\ \tfrac{\text{m}}{\text{s}}$$

The magnitude of the velocity is the vector sum of these components: 16.8 m/s.

$$|\vec{v}| = \sqrt{\left(16.4\ \tfrac{\text{m}}{\text{s}}\right)^2 + \left(3.787\ \tfrac{\text{m}}{\text{s}}\right)^2}\text{F}$$
$$|\vec{v}| = 16.8\ \tfrac{\text{m}}{\text{s}}$$
$$|\vec{v}| \cong 17\ \tfrac{\text{m}}{\text{s}}{}^2$$

Positive *x*- and *y*-components place the velocity vector in the first quadrant. The angle counterclockwise from the positive *x*-axis is found as follows.

$$\tan\theta = \frac{3.787\ \cancel{\tfrac{\text{m}}{\text{s}}}}{16.4\ \cancel{\tfrac{\text{m}}{\text{s}}}}$$
$$\theta = \tan^{-1}\ 0.2309$$
$$\theta \cong 13°$$

(e) The ball strikes the ground at an angle of 38° clockwise from the positive *x*-axis. When the ball strikes the ground, $v_x = 16.4$ m/s, and

$$v_{fy} = v_{iy} + a_y\Delta t$$
$$v_{fy} = \left(7.32\ \tfrac{\text{m}}{\text{s}}\right) + \left(-9.81\ \tfrac{\text{m}}{\text{s}^2}\right)(2.065\ \text{s})$$
$$v_{fy} \cong -13\ \tfrac{\text{m}}{\text{s}}$$

Thus, $\theta = \tan^{-1}\left(\dfrac{12.94\ \cancel{\tfrac{\text{m}}{\text{s}}}}{16.4\ \cancel{\tfrac{\text{m}}{\text{s}}}}\right) = 38°$. A positive *x*- and a negative *y*-component place the resultant vector in the fourth quadrant.

18. The horizontal velocity is 52 m/s. The time of flight is 2.3 s.

$$\Delta y = v_{1y}\Delta t + \frac{1}{2}a_y\Delta t^2$$
$$\Delta y = 0\Delta t + \frac{1}{2}a_y\Delta t^2$$
$$\Delta t^2 = \sqrt{\frac{2\Delta y}{a_y}}$$
$$\Delta t = \pm\sqrt{\frac{2(-27\ \cancel{m})}{-9.81\frac{\cancel{m}}{s^2}}}$$
$$\Delta t = 2.346\ s$$
$$v_x = \frac{\Delta x}{\Delta t}$$
$$v_x = \frac{122\ m}{2.346\ s}$$
$$v_x = 52\ \frac{m}{s}$$

19. Yes, the batter hits a home run. The horizontal and vertical components of the initial velocity are

$$v_x = v_o\cos\theta \quad \text{and} \quad v_{1y} = v_o\sin\theta$$
$$v_x = \left(58\ \frac{m}{s}\right)(\cos 37°) \qquad v_{1y} = \left(58\ \frac{m}{s}\right)(\sin 37°)$$
$$v_x = 46.3\ \frac{m}{s} \qquad v_{1y} = 34.9\ \frac{m}{s}$$

The time required to travel a horizontal distance of 323 m is 7.0 s.

$$v_x = \frac{\Delta x}{\Delta t}$$
$$\Delta t = \frac{\Delta x}{v_x}$$
$$\Delta t = \frac{323\ m}{46.3\ \frac{m}{s}}$$
$$\Delta t \cong 7.0\ s$$

The vertical displacement after 7.0 s is 4.8 m.

$$\Delta y = v_{1y}\Delta t + \frac{1}{2}a_y\Delta t^2$$
$$\Delta y = \left(34.9\ \frac{m}{s}\right)(6.976\ s) + \frac{1}{2}\left(-9.81\ \frac{m}{s^2}\right)(6.976\ s)^2$$
$$\Delta y = 4.8\ m$$

Since this is higher than the fence, the hit will be a home run.

20. (a) The arrow's time of flight will be 7.4 s.

$$T = \frac{2v_i\sin\theta}{g}$$
$$T = \frac{2\left(40.0\ \frac{m}{s}\right)(\sin 65°)}{9.81\ \frac{m}{s^2}}$$
$$T \cong 7.4\ s$$

(b) The arrow's maximum height will be 67 m.

$$H = \frac{v_1{}^2 \sin^2 \theta}{2g}$$

$$H = \frac{\left(40.0\ \frac{\text{m}}{\text{s}}\right)^2 (\sin^2 65^\circ)}{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}$$

$$H \cong 67\ \text{m}$$

(c) The range of the arrow will be 1.2×10^2 m.

$$R = \frac{v_1{}^2 \sin 2\theta}{g}$$

$$R = \frac{\left(40.0\ \frac{\text{m}}{\text{s}}\right)^2 [\sin 2(65^\circ)]}{\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}$$

$$R \cong 1.2 \times 10^2\ \text{m}$$

(d) At 2.0 s, the horizontal position is 34 m and the vertical position is 53 m.

$$\Delta x = v_x \Delta t$$

$$\Delta x = \left(40.0\ \frac{\text{m}}{\text{s}}\right)(\cos 65^\circ)(2.0\ \text{s})$$

$$\Delta x \cong 34\ \text{m}$$

$$\Delta y = v_{1y}\Delta t + \frac{1}{2} a_y \Delta t^2$$

$$\Delta y = \left(40.0\ \frac{\text{m}}{\text{s}}\right)(\sin 65^\circ)(2.0\ \text{s}) + \frac{1}{2}\left(-9.81\ \frac{\text{m}}{\text{s}^2}\right)(2.0\ \text{s})^2$$

$$\Delta y \cong 53\ \text{m}$$

(e) At 6.0 s, the horizontal velocity is 17 m/s and the vertical velocity is −23 m/s.

$$v_x = v_o \cos \theta$$

$$v_x = \left(40.0\ \frac{\text{m}}{\text{s}}\right)(\cos 65^\circ)$$

$$v_x = 17\ \frac{\text{m}}{\text{s}}$$

$$v_{fy} = v_{1y} + a_y \Delta t$$

$$v_{fy} = \left(40.0\ \frac{\text{m}}{\text{s}}\right) \sin 65^\circ + \left(-9.81\ \frac{\text{m}}{\text{s}^2}\right)(6.0\ \text{s})$$

$$v_{fy} \cong -23\ \frac{\text{m}}{\text{s}}$$

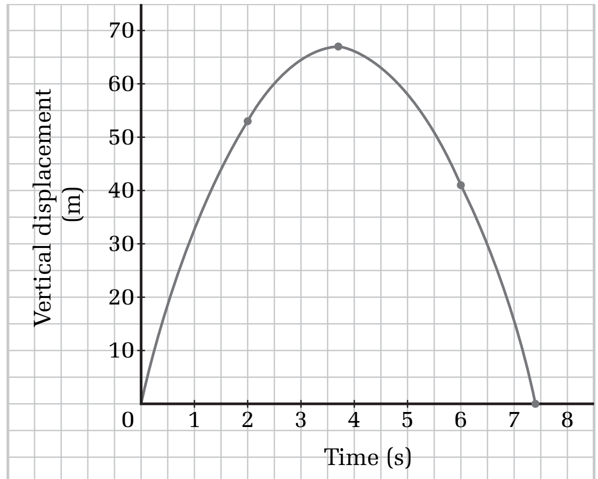
(f) The direction of the arrow after 6.0 s was 53° clockwise from the positive *x*-axis.

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

$$\theta = \tan^{-1}\left(\frac{22.6\ \cancel{\frac{\text{m}}{\text{s}}}}{16.9\ \cancel{\frac{\text{m}}{\text{s}}}}\right)$$

$$\theta \cong 53^\circ$$

A positive x-component and a negative y-component place the resultant vector in the fourth quadrant.



21. (a) The magnitude of the change in velocity is 2.1 m/s. The direction of the change in velocity is always toward the centre of the circular path of the ball.

$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$$
$$|\Delta\vec{v}| = \sqrt{|\Delta\vec{v}_2|^2 + |\Delta\vec{v}_1|^2 - 2|\Delta\vec{v}_2||\Delta\vec{v}_1|\cos 40°}$$
$$|\Delta\vec{v}| = \sqrt{\left(3.0\tfrac{\text{m}}{\text{s}}\right)^2 + \left(3.0\tfrac{\text{m}}{\text{s}}\right)^2 - 2\left(3.0\tfrac{\text{m}}{\text{s}}\right)\left(3.0\tfrac{\text{m}}{\text{s}}\right)\cos 40°}$$
$$|\Delta\vec{v}| = 2.1\tfrac{\text{m}}{\text{s}}$$

(b) The acceleration of the ball is $1.2\ \text{m/s}^2$ toward the centre of the circular path of the ball.

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$
$$\vec{a} = \frac{2.052\tfrac{\text{m}}{\text{s}}}{1.75\ \text{s}}$$
$$\vec{a} \cong 1.2\ \tfrac{\text{m}}{\text{s}^2}\ \text{toward the centre of circle}$$

22. (a) The acceleration of the electron is $1.33 \times 10^{14}\ \text{m/s}^2$.

$$a_c = \frac{v^2}{r}$$
$$a_c = \frac{\left(2.00 \times 10^7\ \tfrac{\text{m}}{\text{s}}\right)^2}{3.00\ \text{m}}$$
$$a_c \cong 1.33 \times 10^{14}\ \tfrac{\text{m}}{\text{s}^2}$$

(b) The centripetal force on the electron is 1.21×10^{-16} N.

$$F_c = \frac{mv^2}{r}$$
$$F_c = \frac{\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(2.00 \times 10^7\ \tfrac{\text{m}}{\text{s}}\right)^2}{3.00\ \text{m}}$$
$$F_c \cong 1.21 \times 10^{-16}\ \text{N}$$

23. The centripetal force required to keep the car on the curve in the road is 4.9×10^3 N.

$$F_c = \frac{mv^2}{r}$$

$$F_c = \frac{(1500\text{ kg})\left[\left(65\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}}\right)\left(\frac{1000\text{ m}}{\cancel{\text{km}}}\right)\left(\frac{\cancel{\text{h}}}{3600\text{ s}}\right)\right]^2}{1.0 \times 10^2\text{ m}}$$

$$F_c \cong 4.9 \times 10^3\text{ N}$$

If the centripetal force is provided by friction between the tires and the road, the coefficient of friction must be 0.33.

$$F_f = F_c$$

$$\mu mg = F_c$$

$$\mu = \frac{F_c}{mg}$$

$$\mu = \frac{4890\text{ N}}{(1500\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}$$

$$\mu \cong 0.33$$

24. If the banked curve must provide all of the centripetal force, the car must travel at 8.9 m/s or 32 km/h.

$$\tan\theta = \frac{v^2}{rg}$$

$$v = \sqrt{rg\tan\theta}$$

$$v = \sqrt{(75.0\text{ m})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\tan 6.2^\circ}$$

$$v = 8.9\frac{\text{m}}{\text{s}}\ \text{ or } 32\ \frac{\text{km}}{\text{h}}$$

25. If the banking of the curve provides all of the centripetal force, the angle of banking must be 3.7°.

$$\tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

$$\theta = \tan^{-1}\left(\frac{\left(15\ \frac{\text{m}}{\text{s}}\right)^2}{(350\text{ m})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}\right)$$

$$\theta = \tan^{-1} 0.06553$$

$$\theta \cong 3.7^\circ$$

26. At the minimum speed, the centripetal force is provided entirely by the force of gravity. The minimum speed that will make the motorcycle remain on the vertical circular track is 9.9 m/s.

$$F_c = F_g$$

$$\frac{\cancel{m}v^2}{r} = \cancel{m}g$$

$$v = \sqrt{rg}$$

$$v = \sqrt{(10.0\text{ m})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}$$

$$v \cong 9.9\ \frac{\text{m}}{\text{s}}$$

27. The coefficient of friction must be 0.62. To prevent the riders from sliding down the wall, the force of friction between the riders and the wall must equal the force of gravity. The normal force of the wall on the riders is the centripetal force.

$$F_f = F_g$$
$$\mu F_N = mg$$
$$F_N = \frac{mv^2}{r}$$
$$\mu\frac{\cancel{m}v^2}{r} = \cancel{m}g$$

$$\mu = \frac{rg}{v^2}$$
$$\mu = \frac{rg}{\left(\frac{2\pi r}{T}\right)^2}$$
$$\mu = \frac{(2.5\text{ m})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}{\left(\frac{2\pi(2.5\text{ m})}{2.5\text{ s}}\right)^2}$$
$$\mu = 0.62$$

28. **(a)** Your speed if you were standing in the town square of Quito, Ecuador, would be 464 m/s.

$$v = \frac{\Delta d}{\Delta t}$$
$$v = \frac{2\pi r}{T}$$
$$v = \frac{2\pi(6.38 \times 10^6\text{ m})}{(1.0\ \cancel{\text{day}})\left(24\ \frac{\cancel{\text{h}}}{\cancel{\text{day}}}\right)\left(60\ \frac{\cancel{\text{min}}}{\cancel{\text{h}}}\right)\left(60\ \frac{\text{s}}{\cancel{\text{min}}}\right)}$$
$$v \cong 464\ \frac{\text{m}}{\text{s}}$$

(b) The centripetal force in newtons needed to keep you on a circular path with a radius equal to Earth's radius is 0.0336 times your mass. (**Note:** Some students might be more comfortable if a numerical value was used for mass. For example, if a student's mass is 55 kg, the required centripetal force would be 185 N.)

$$F_c = \frac{mv^2}{r}$$
$$F_c = \frac{m\left(463.97\ \frac{\text{m}}{\text{s}}\right)^2}{6.4 \times 10^6\text{ m}}$$
$$F_c = 0.0336m\text{ N}$$

(c) The centripetal force is provided by the force of gravity, and is directed toward the centre of Earth.

(d) Your weight in newtons is 9.81 times your mass. (If a student's mass is 55 kg, the student's weight is 540 N.)

(e) Earth exerts a normal force on you equal in magnitude and opposite in direction to the force you exert on Earth's surface.

(f) You are not thrown off Earth's surface because only $0.0336m$ N is required to keep you in the circular path, and gravity provides $9.81m$ N. The ratio of the amount of force provided by gravity and the amount of centripetal force needed to keep you on Earth's surface is $\frac{9.81m\text{ N}}{0.0336m\text{ N}} = 292$.